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## METHOD FOR DETERMINING THE TIME CONSTANTS CHARACTERIZING THE INTENSITY OF STEEL MIXING IN CONTINUOUS CASTING TUNDISH

## METODA WYZNACZANIA STAŁYCH CZASOWYCH CHARAKTERYZUJĄCYCH INTENSYWNOŚĆ MIESZANIA SIĘ STALI W KADZI POŚREDNIEJ URZĄDZENIA COS

A common method used in identification of hydrodynamics phenomena occurring in Continuous Casting (CC) device's tundish is to determine the RTD curves of time. These curves allow to determine the way of the liquid steel flowing and mixing in the tundish. These can be identified either as the result of numerical simulation or by the experiments - as the result of researching the physical models. Special problem is to objectify it while conducting physical research. It is necessary to precisely determine the time constants which characterize researched phenomena basing on the data acquired in the measured change of the concentration of the tracer in model liquid's volume. The mathematical description of determined curves is based on the approximate differential equations formulated in the theory of fluid mechanics. Solving these equations to calculate the time constants requires a special software and it is very time-consuming. To improve the process a method was created to calculate the time constants with use of automation elements. It allows to solve problems using algebraic method, which improves interpretation of the research results of physical modeling.

*Keywords:* physical modelling, flow of fluids, RTD curves

Powszechnie stosowaną metodą w identyfikacji zjawisk hydrodynamicznych, zachodzących w kadzi pośredniej urządzenia COS, jest wyznaczanie krzywych czasu rezydencji RTD. Krzywe te umożliwiają określenie sposobu przepływu i mieszania się ciekłej stali w kadzi pośredniej. Mogą one być wyznaczane zarówno metodami numerycznymi jak i w sposób eksperymentalny, jako wynik badań na modelach fizycznych. Szczególny problem stanowi ich obiektywizacja w przypadku badań fizycznych. Konieczne jest wówczas precyzyjne wyznaczenie stałych czasowych charakteryzujących badane zjawiska na podstawie danych uzyskanych z pomiaru zmiany stężenia znacznika w objętości cieczy modelowej. Matematyczny opis wyznaczonych krzywych opiera się na złożonych równaniach różniczkowych, sformułowanych w teorii mechaniki płynów. Rozwiązywanie tych równań, w celu wyznaczenia wspomnianych stałych czasowych, wymaga specjalistycznego oprogramowania oraz jest czasochłonne. Aby usprawnić ten proces opracowano sposób wyznaczania stałych czasowych z wykorzystaniem metody elementów automatyki. Umożliwia ona rozwiązywanie problemu metodami algebraicznymi, co zdecydowanie usprawnia interpretację uzyskiwanych wyników badań modelowania fizycznego.

### 1. Introduction

The proper understanding of flow and mixing processes of liquid steel in tundish of CC machine needs the knowledge about problems concerning fluid mechanics. Such knowledge is important both for the suitable formulation of research methodology (e.g. modelling research) and for analysis of the results concerning the dynamics of steel movement in tundish. From the definition of fluid mechanics deals with analysis of fluid movement, equilibrium states, their interaction to the limiting walls and bodies immersed in those liquids.

For the description of the physical phenomena occurring in the nature fluid mechanics applied models, that reflect the fluid movement, written in the form of mathematical equa-

tions. Assumptions for such equations contain some simplifications which enable to solve them. One of such simplifications is assumption taken in the description of features characterizing the liquids: volume of the fluid is changing insignificantly under the influence of external forces. Thus, in calculations fluids are treated as an incompressible bodies (law of constant density  $\rho = M/V = \text{const.}$ ). To the other simplification belongs assumption that liquid is a continuous medium. Such simplification relies on not taking into consideration the molecular structure of the studied medium and disordered movement of molecule [1]. Taking into account such simplifications four theoretical models concerning fluid movement are distinguished [2]:

- perfect fluid (viscous and incompressible),
- viscous and incompressible fluid,

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- inviscid and compressible fluid,
- real fluid (viscous and compressible).

In research of liquid steel flow through tundish steel is treated as a real fluid (viscous). The fundamental law of real fluid mechanics is hydrodynamic Newton's law. It says that shearing stress  $\tau_s$  in fluid is directly proportional to the shearing speed  $\dot{\gamma}$ , whereas the factor of proportionality called viscosity  $\eta$  is a characteristic parameter for the given kind of fluid. Such law can be written in the following form:

$$\tau_s = \dot{\gamma} \cdot \eta \quad (1)$$

According to the basic statement of fluid mechanics that every liquid is behaving in agreement with law of mass and momentum conservation and hypothesis of continuity, the movement equations of Newtonian compressible liquid at the constant value of viscosity coefficient were formulated. Such equations are known as Navier-Stokes's equations and can be written in the following form [1, 3]:

$$\begin{aligned} \frac{dv_x}{dt} &= X - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\eta}{\rho} \nabla^2 v_x + \frac{1}{3\rho} \frac{\partial}{\partial x} \text{div } v \\ \frac{dv_y}{dt} &= Y - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\eta}{\rho} \nabla^2 v_y + \frac{1}{3\rho} \frac{\partial}{\partial y} \text{div } v \\ \frac{dv_z}{dt} &= Z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\eta}{\rho} \nabla^2 v_z + \frac{1}{3\rho} \frac{\partial}{\partial z} \text{div } v \end{aligned} \quad (2)$$

where:  $p$  – pressure, Pa

$v$  – speed,  $\text{m}\cdot\text{s}^{-1}$

$t$  – time, s

$\eta$  – dynamic viscosity,  $\text{Pa}\cdot\text{s}$

$\rho$  – density,  $\text{kg}\cdot\text{m}^{-3}$

$X, Y, Z$  – Cartesian coordinates.

$x, y, z$  – spatial coordinates

To obtain solution of steel flow in tundish the closest to the real one the system of Navier-Stokes's equations should be complemented by the equation describing phenomena connected with fluid turbulence. Mostly in that case the two-equation model  $k\text{-}\varepsilon$  [3-6] is applied.

## 2. Theoretical basis of RTD curves defining

When the character of steel flow through tundish of CC machine is considered, it is assumed that such reactor fulfill the condition of continuous reactor. Taking into account the kind of flow in working zone of tundish three basic areas are distinguished:

- area of well mixed flow (turbulent flow) – in the inlet part of the tundish,
- area of dispersed plug flow (laminar) – in the channel part of tundish,
- area of dead flow (stagnant) occurring in different parts of tundish depending on its construction.

The first two kinds of flow, considering the correct tundish working, are desirable, whereas the third one is harmful and the range of its occurring should be minimal.

Area of well mixed flow influence the proper course of homogenization process (both chemically and taking into account temperature) of liquid steel in the tundish. The range

of such area does not include too small volume of tundish, because it will be not sufficient to the efficient mixing. Too high mixing intensity can cause the secondary contamination of the steel by the nonmetallic impurities from the slag phases covering the surface of melt and also the erosion of refractories. In extreme cases, when the turbulence is very high, the continuity of slag can be broken and as a consequence the secondary oxidation of the steel melt is observed.

In the area of dispersed plug flow steel mixing is not observed, the role of such area is to ensure optimal conditions for the height and floating the nonmetallic inclusions to the slag. The range of such area should cover all outlets from tundish to the moulds – in that way the precise dosing of steel is possible.

Occurrence of areas of well mixed flow and dispersed plug flow in the working zone of tundish depends on the specific constructional features of the tundish and the direction of the steel flux using flow devices control.

During the regulation of steel flow in tundish, if the area of dispersed plug flow will be increased, then the area of well mixed flow is decreased and vice versa. Thus, the choice of proper equipment of working zone of tundish is always a results of compromise coming from necessity of choosing the proper proportion of described areas for given conditions occurring in particular steel works. In industrial tundishes the participation of well mixed flow area is in the range form 40 to 60%, whereas the participation of dispersed plug flow is from 10 to 30%.

When the area of dead flow is considered, there is no such problem as in case of well mixed and dispersed plug flow; this is why dead flow negatively influences the flow and mixing of steel in tundish, so always such area should be maximally limited. Optimization of tundishes working considering steel flow needs to conduct many researches. Quantitative results of research are obtained as an effect of determination the RTD curves (Residence Time Distribution) [7-10]. In modeling research RTD curves are determined by registering the changes of tracer concentration in water as a function of time. Theoretical basis of RTD characteristics has the source in the inert function of age distribution, which assume that in the period between  $t$  and  $\Delta t$  the fraction of substance being in reactor equals the product of  $I(t) \cdot \Delta t$ .  $I(t)$  is a continuous function and after arrangement the relationship can be written:

$$\int_0^{\infty} I(t) dt = 1 \quad (3)$$

Assuming that:

- reactor is in equilibrium state,
- transport of fluid at the inlet and outlet has advective character – convection of substance by the flowing liquid is cause in that way that speed of rising substance equals the speed of flowing liquid,
- liquid is incompressible,

such function for the distribution of residence time of liquid particle removing the continuous reactor can be written in the following form:

$$\int_0^{\infty} E(t) dt = 1 \quad (4)$$

The higher notation means that the particle of fluid stays in reactor in time  $t$ . If the particle of fluid removes the reactor in time shorter than  $t$ , the equation can be written in the form:

$$\int_0^t E(t) dt = F(t) \quad (5)$$

Such function is called cumulative function of distribution ( $F(t)$ ), popularly  $F$ . When the particle of fluid stays in reactor in the time longer than  $t$ , then:

$$\int_0^{\infty} E(t) dt = 1 - \int_0^t E(t) dt \quad (6)$$

Function  $E(t)$  and  $F(t)$  are the most common RTD functions used to describe the character of steel flow in tundish of CC machine [5, 6, 11-17].

Practically the determination of RTD curves in physical models of tundishes is done in such way, that on the inlet of tundish model tracer is introduced, which change of concentration is measured on the particular outlets.

### 3. Methods of RTD curves determination

Its physical interpretation of the RTD characteristics E-type is the following: if to the vessel with  $V$  volume and  $Q$  volumetric flow the tracer with very small mass  $M$  is injective introduced at the inlet (assuming the enough long average time of residence  $t_{sp}$ ), the change of this tracer concentration is registered on the outlets as a function of time  $C(t)$ , then the distribution of dimensionless time residence of tracer can be written in the following form:

$$E(t) = \frac{C(t)}{\int_0^{\infty} C(t) dt} \quad (7)$$

Fig. 1 presents the chosen characteristics of residence time distribution of tracer obtained by impulse method. Designations in Fig. 1 (case A to C) concern different variants of working zone equipment (flow control device – turbulent inhibitors) of the tundish. Such kind of curve in modelling research of liquid steel flow through tundish is called RTD curve E – type (from the type of function, which describes it).

The second method of determining the RTD curves is called step one. The course of experiment relies on the step function of tracer concentration distribution at the inlet from value 0 to  $C_{max}$  and measuring the changes of its concentration at the tundish model outlets. Such method is willingly used in modelling the steel flow through tundish due to fact that it directly represents the technique of sequence casting. Results obtained in such way are described by function  $F$  which has the following form:

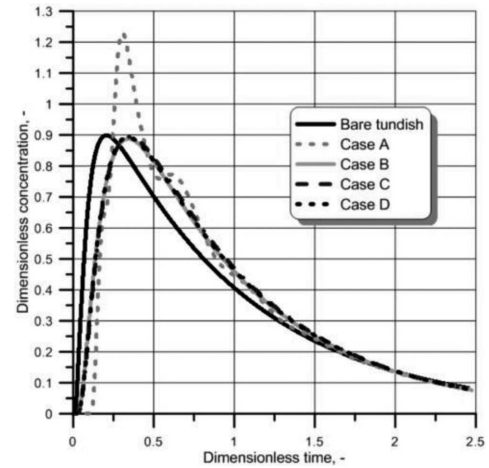


Fig. 1. Example of RTD characteristic – E-type [13]

$$F(t) = \frac{C(t)}{C_{max}} \quad (8)$$

where:

$C(t)$  – change of tracer concentration in modelling liquid as a function of time,

$C_{max}$  – tracer concentration.

Fig. 2 shows the chosen characteristics of residence time distribution of tracer obtained by step method. This type of curve in modelling research of liquid steel flow through tundish is called RTD curve F – type [18]. Designations in Fig. 1 (case A to C) concern different variants of working zone equipment (flow control device – turbulent inhibitors) of the tundish.

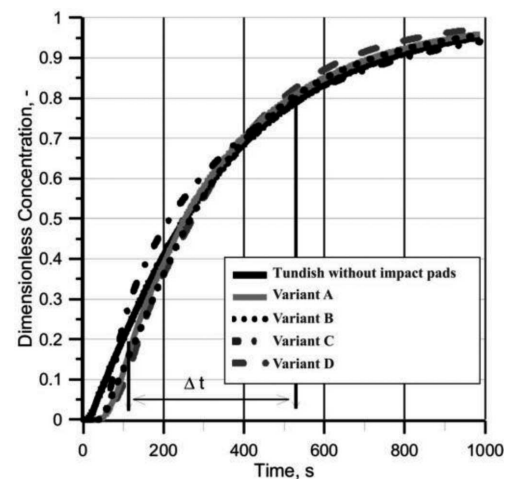


Fig. 2. Example of RTD characteristic – F-type [18]

In research practice curves F-type are good measure of the character of steel mixing in tundish, whereas curves E – types are used to the quantitative determination of the flows kinds in tundish area [19-23].

### 4. Method of determination of time constants $\tau$ and $T_0$

Method of determination of time constants  $\tau$  and  $T_0$  is applied in analysis of RTD curves F-type. Such curves are obtained from the modelling research, as a time response on

the step input of tracer concentration changes in the modelling liquid (water, steel). When that method was worked out different types of linear elements of automatics were considered – they should the best represent the character of sequence steel continuous casting. It was stated that the course of described process can be shown in the operator way by the transition function corresponding to the inert element of 1-st order. Fig. 3 shows the elementary leap of input signal  $u(t) = 1(t)$  and the time response of the inert element of 1-st order [24-28].

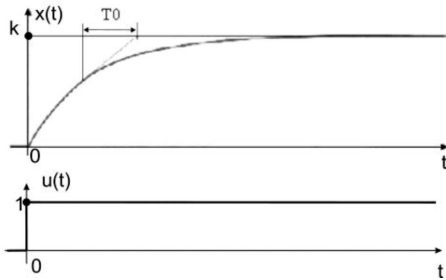


Fig. 3. Output signal of the inert element of 1-st order as a response to the elementary leap of input signal [26]

Transmittance of inert element of I order (as a ratio of output to input  $x(s)/u(s)$ ) in the domain of the operator can be written in the form:

$$G(s) = \frac{k}{1 + sT0} \tag{9}$$

Time constant  $T0$  (in seconds) can be determined graphically drawing tangent to the response  $x(t)$  – see Fig. 3. If the time constant will approach zero, the answer approach the ideal simulation of input signal, so the step change in time at the output. Thus, intensification  $k$  and also time constant  $T0$  characterize in the same time static and dynamic features of this element.

Taking into account that output holes of tundish are distant form the inlet point, thus the delaying element should be considered. Fig. 4 presents the response of delaying element to the elementary leap at the input [26].

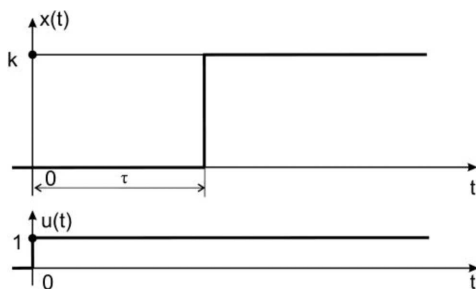


Fig. 4. Step response to the delaying element [26]

If the elementary leap at the input is written as  $u(t) = 1(t)$ , at the output of the delaying element with the delaying time  $\tau$ , will be signal:

$$x(t) = k \cdot 1(t - \tau) \tag{10}$$

So it will be  $k$  time strengthen and delayed about  $\tau$  considering input signal. Taking into account domain of operator, the

transmittance of the delaying element can be obtained in the following form:

$$G(s) = ke^{-s\tau} \tag{11}$$

Obtained time characteristics (from modelling research) – see Fig. 2 – enable to assume that the model describing the dynamics of liquid mixing in tundish is inert system with delay about transmittance:

$$G(s) = \frac{ke^{-s\tau}}{1 + sT0} \tag{12}$$

which is a series connection of mentioned above basic elements: inert element and delaying element. Fig. 5 shows graphically time characteristics of element with the transmittance (12) and the interpretation of constants occurring in equation (12).

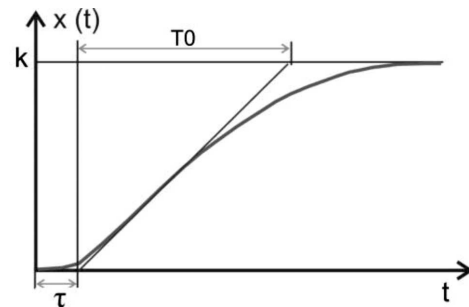


Fig. 5. Typical step response of inert system with delay about transmittance (12) [26]

Physical interpretation of described time constants (in seconds) for the studied system is following:

$\tau$  – time of tracer coming from the inlet point to the outlet of the tundish,

$T0$  – water mixing time till obtaining the tracer concentration determined by  $k$  strengthen.

Using the time characteristics presented in Fig. 5 it can be assumed that  $k$  strengthen of the studied inert system connected with tracer mixing in the tundish model equals 1. The rest constants ( $\tau$ ,  $T0$ ) of characteristics (12) are determined using the least squares method basing on measuring data of tracer concentration changes in water registered in equal time periods  $\Delta t = const$ . If the initial values of concentration are defined as  $x_1, x_2, \dots, x_n$ , then to determine constants  $\tau$  and  $T0$  the problem of optimization should be solved:

$$\min k, \tau, T0 \sum_i = 1^n (x_i - x_i^*(k, \tau, T0))^2 \tag{13}$$

where:  $x_i^*(k, \tau, T0)$  – time form of function corresponding to the transmittance  $G(s)$  described by the relationship (12) for the given values of constants  $k$ ,  $\tau$  and  $T0$  in time  $t = i \cdot \Delta t$ .

$$x_i^*(k, \tau, T0) = L^{-1}(G(s)) \quad \left| \begin{array}{l} k, \tau, T0 \\ t = i \cdot \Delta t \end{array} \right. \tag{14}$$

where:  $L^{-1}(G(s))$  – inverse Laplace transform from function  $G(s)$  [26-28].

Fig. 6 presents the example of conducted optimization results.

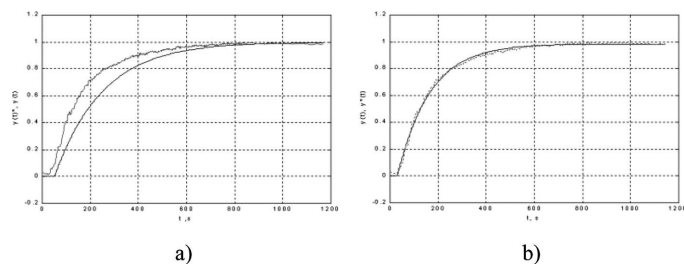


Fig. 6. Example of estimation course: a). measuring results and choice of primary values of time constant, b) estimation results [19]

## 5. Summary

The worked out method of interpretation of RTD curves F-type (obtained from modelling research) enables to determine easily and quickly the hydrodynamic conditions occurring in the tundish. It is commonly used to determine the hydrodynamics of liquid steel mixing in particular outlet areas of tundishes and to determine the range of occurrence of the transient zone during casting two different grades of steel differing with chemical composition in one sequence (casting with transient zone).

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