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INFLUENCE OF PNM-0.38PT DEFECT ON TRANSMISSION OF MULTILAYER PERIODIC PHONONIC STRUCTURE

In this work, the impact of the defect on the transmission of a mechanical wave in a periodic quasi-one-dimensional structure was investigated. The multilayer structure was made of PLA and air, while the defect layer was PNM-0.38PT with a significantly higher value of acoustic impedance in relation to the materials of the base structure. The influence of the position of the defect in the structure and its thickness was analysed. Transmission as a function of frequency was determined using the Transfer Matrix Method algorithm. The work showed the presence of band gaps in the analyzed structures. The influence of the symmetry of structures and substructures on the transmission of a mechanical wave was investigated. The influence of the number of layers with very low acoustic impedance (air) on the number of high transmission peaks with a small half-width was also demonstrated.

Keywords: Phononic structures; band gap; defect; transfer matrix

1. Introduction

More than a hundred years ago, structures characterized by translational invariance were already analyzed [1]. The repetition of the unit cell in the structure affects the propagation of medium disturbances such as mechanical waves [2], but also for electromagnetic waves [3], plasmonic [4] or thermal [5]. Designing the right structure allows to achieve phenomena such as ‘invisibility’ [6], sub-wavelength imaging [7], or bending waves in the ‘wrong’ direction [8]. Mechanical waves are scattered on phononic structures [9] and electromagnetic waves on photonic structures [10]. When the size of the lattice constant is comparable to the wavelength, therefore, interference on metaatoms (elements of the structure) significantly affects on the propagation of mechanical waves.

The phononic structures consist of mechanical wave dissipation elements embedded in the matrix material. A mechanical wave (acoustic or elastic) interacting with elements of the phononic structure due to diffraction and destructive interference may, for given frequency ranges, not propagate in the structure. This phenomenon is called the Phononic Band Gap (PhnBG) [11-13]. It allows the manipulation of mechanical waves, and

thus the design of devices such as, for example, filters [14,15], microfabricated acoustic crystal slabs [16], acoustic cloaking [17], lenses for the mechanical waves [18], waveguides [19], phononic multilayer sensors [20], acoustic diodes [21], noise suppressors [22], barriers for acoustic waves [23], or used as energy harvesters [24-26].

Many works focus on obtaining a bandgap with given properties. In the work [27] Alagoz et al. investigated the focal point frequency dependence for a sonic crystal lens with a triangular lattice configuration Bilal and Hussein in [28] using a genetic algorithm, they optimized the structure to obtain a wide bandgap. In the works [29,30], the authors also used a genetic algorithm to minimize wave transmission. In works [31,32] the authors optimized the structure in order to maximize reflectance. In the work [33], the authors studied the effect of layer thickness on the transmission of structures made of amorphous materials. The occurrence of the bandgap phenomenon was also presented in [34,35]. In the work [11], the occurrence of characteristic transmission peaks occurring in phononic structures was demonstrated. Many works in the field of analysis of the properties of phononic structures focus on optimizing the spatial distribution of materials in the structure [36-40].

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The paper analyzes the influence of a defect in a multilayer structure with a thickness of a single, double and triple layer on the transmission of a mechanical wave in a periodic structure.

2. Research

To analyze the properties of phononic structures in this work, the transmission matrix method (TMM) for quasi-one-dimensional (1D) structures were used. The simulations did not take into account the energy absorption in materials to analyze only the effect of the spatial distribution of materials on wave propagation. Absorption reduces the intensity of the wave without affecting the frequency response.

Using the TMM algorithm, it is possible to determine the frequency range of the bandgap [41], [42].

Propagation of pressure p disturbance in the medium at time t in a multilayer medium, where the phase velocity c_i of mechanical wave propagation in the layer i for a given material, is determined by the equation

$$\frac{1}{c_i^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0 \quad (1)$$

and its solution for the one-dimensional case takes the form

$$p_i = C_i(x) e^{-j\omega t} \quad (2)$$

The C_i is defined in equation (3) where A_i corresponds to the amplitude of the wave propagating in the structure in the direction of the incident mechanical wave, and B_i is the amplitude of the wave propagating in the opposite direction.

$$C_i = A_i e^{jk_i x} + B_i e^{-jk_i x} \quad (3)$$

The k_i is wave vector, where ω is a circular frequency, the f is frequency, which is defined as

$$k_i = 2\pi f / c_i = \omega / c_i \quad (4)$$

The mechanical wave with amplitude p_{in}^+ , falling on the multilayer structure of x_i layers which are built of A and B materials with the thickness of a given layer defined by d_i . The p_{out}^+ is transmitted wave amplitude and p_{in}^- is reflected. Wave propagation in a quasi one-dimensional structure is described by the following matrix equation

$$\begin{bmatrix} p_{in}^+ \\ p_{in}^- \end{bmatrix} = M \begin{bmatrix} p_{out}^+ \\ 0 \end{bmatrix} \quad (5)$$

where M is a characteristic matrix of a structure consisting of N layers, and is defined as

$$M = D_{in,1} \left[\prod_{i=1}^{N-1} P_i D_{i,i+1} \right] P_N D_{N,out} \quad (6)$$

The propagation matrix P_i inside a given layer i which has a thickness is d_i is determined by

$$P_i = \begin{bmatrix} e^{jk_i d_i} & 0 \\ 0 & e^{-jk_i d_i} \end{bmatrix} \quad (7)$$

The transmission matrix $\Phi_{i,i+1}$ between i and $i+1$ layers is defined as

$$\Phi_{i,i+1} = \frac{1}{t_{i,i+1}} \begin{bmatrix} 1 & r_{i,i+1} \\ r_{i,i+1} & 1 \end{bmatrix} \quad (8)$$

The Fresnel transmission coefficient $t_{i,i+1}$ and reflectance coefficient $r_{i,i+1}$ are given by

$$t_{i,i+1} = \frac{2c_{i+1}\varrho_{i+1}}{c_{i+1}\varrho_{i+1} + c_i\varrho_i} \quad (9)$$

$$r_{i,i+1} = \frac{c_{i+1}\varrho_{i+1} - c_i\varrho_i}{c_{i+1}\varrho_{i+1} + c_i\varrho_i} \quad (10)$$

In equations (9) and (10) for the layer i material the c_i is phase velocity and ϱ_i is mass density.

The transmission T through the multilayer structure of the mechanical wave is determined as

$$T = \frac{Z_{out}}{Z_{in}} \left| \frac{1}{M_{11}} \right|^2 \quad (11)$$

$$T + R = 1 \quad (12)$$

In lossless structures, transmission T and reflectance R are related by equation (12).

To analyze the effect of the defect on the transmission of the mechanical wave, the selected layers of the BABABABAB structure were replaced with the defect material PNM-0.38PT (0.2 mol% Fe-doped relaxor-based ferroelectric 0.62PB (MG_{1/3}NB_{1/3})O₃-0.38PbTiO₃) which was marked as F. This material was characterized by a much higher value of acoustic impedance (approx. 13 times) than the used PLA (polylactic acid marked as material B) or air (approx. 83.5 thousand times) which are marked as material A. The material parameters used in the work are listed in TABLE 1. The thickness of all was 1 mm.

TABLE 1

Density ρ and speed of sound c for the materials used [43-45]

Material	Layer mark	ρ [kg/m ³]	c [m/s]
Air	A	1.29	331,45
PLA	B	1240	2220
PNM-0.38PT	F	8093	4410

Three cases were analyzed in the paper. In the first one (Fig. 1) there was a single defect layer replacing successive layers in the analyzed structure. The second case (Fig. 2) occurred when the defect involved two adjacent layers. In the third one (Fig. 3), the transmission was determined for the case of three adjacent layers of the defect material. The introduction of the defect layer in all cases reduced the total transmission of the mechanical wave

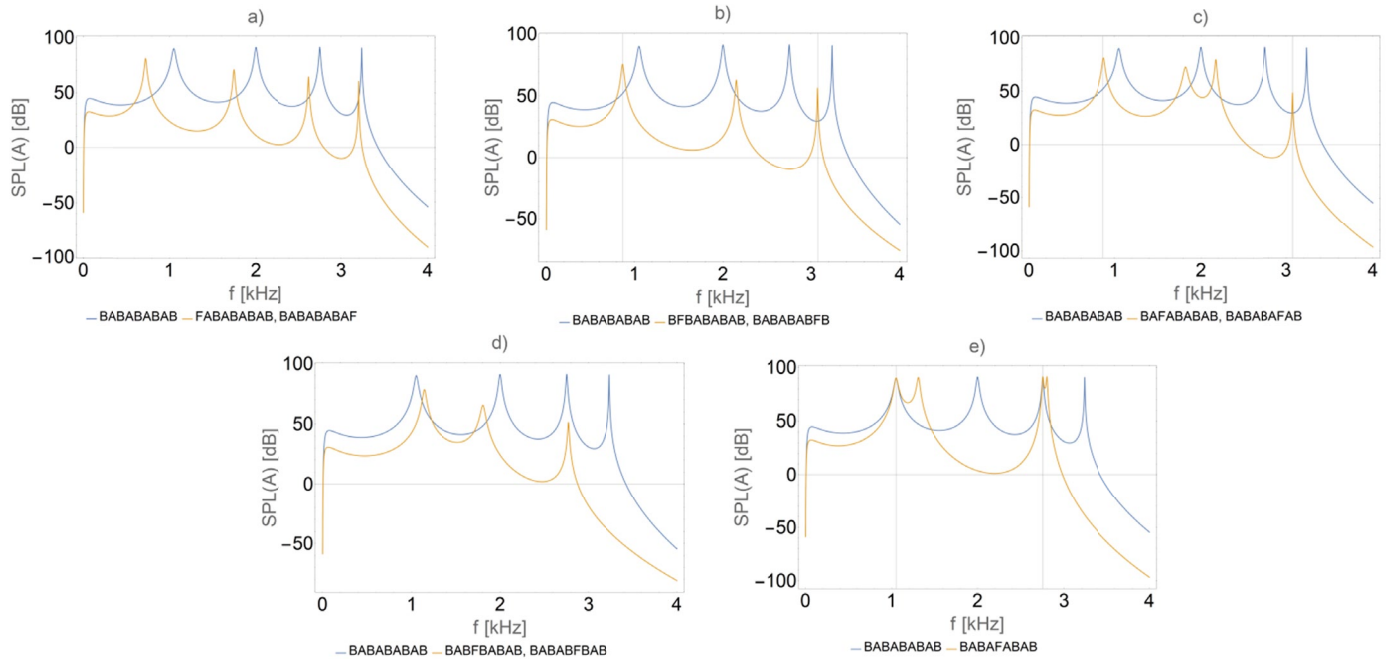


Fig. 1. Influence of a single layer thickness defect on mechanical wave transmission

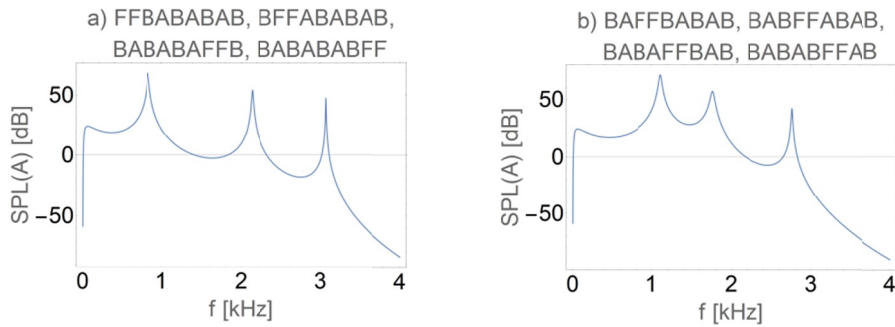


Fig. 2. Effect of a defect with a thickness of two layers on the transmission of a mechanical wave

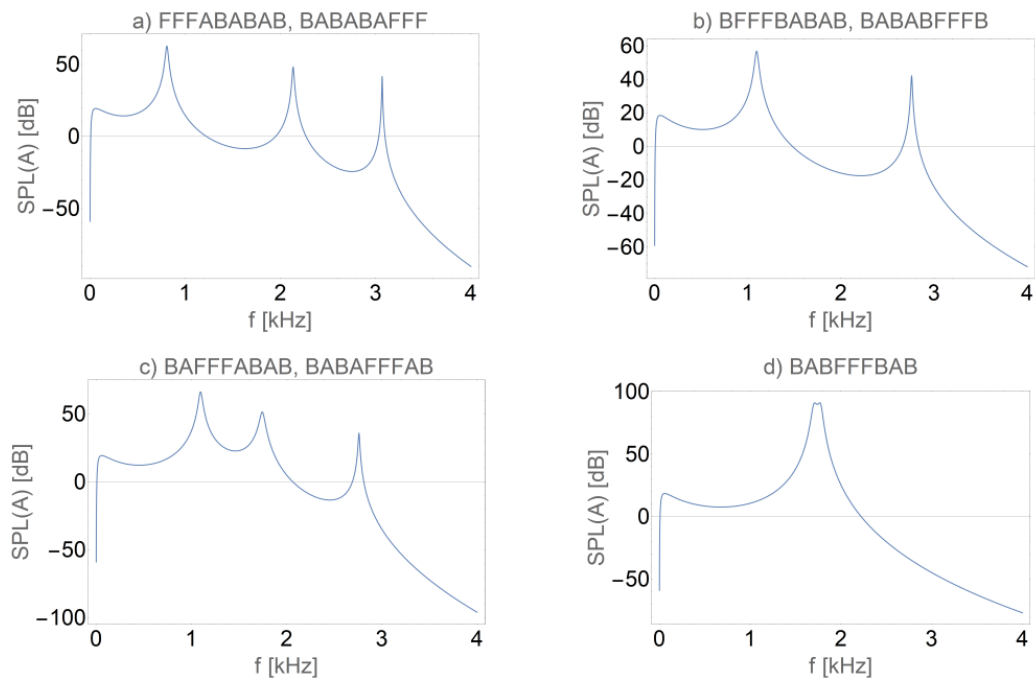


Fig. 3. Influence of a defect with a thickness of three layers on the transmission of a mechanical wave

defined as the transmission integral in the investigated frequency range. A reduction of 30 dB means a reduction of wave energy by 99.9%. The observed decrease in transmission in some of the frequency ranges below 30 dB due to the introduction of defects from a material with a significantly higher acoustic impedance means the occurrence of the phononic band gap phenomenon. The number of transmission peaks in the spectrum corresponded to the number of layers consisting of air (material A). As can be seen from Figs. 1a-d, Fig. 2 and Figs. 3a-c, the transmission for symmetrical structures is the same.

When a single defect layer was present on the extreme layers of the structure (Fig. 1a-c), a shift of all high transmission peaks towards lower frequencies is observed. For the structures shown in Fig. 1b and Fig. 1c, the occurrence of characteristic transmission peaks was observed for the same frequencies marked in the Figures. They occurred at the frequencies of 868 Hz and 3074 Hz. In Fig. 1e, there were common high transmission peaks for the base structure and the structure with a defect for the frequencies of 1051 Hz and 2753 Hz.

In the case of a defect replacing two layers (Fig. 2), two transmission characteristics were observed, each with three peaks (there were three layers of material A in the structures). The same nature of the transmission may be caused by the presence of BBF and FBB substructures in the symmetrical structures occurring between the air layers (material A).

Increasing the defect to triple the layer thickness (Fig. 3) resulted in a further decrease in transmission. Placing the defect in the central place of the structure (Fig. 3d) resulted in the formation of two transmission peaks located in a very close frequency range to each other, merging into a transmission band with a larger half-width, which can be used as a bandpass filter.

3. Conclusions

The study analyzed the influence of PNM-0.38PT defect on transmission of multilayer periodic phononic structure. It was shown that the symmetrical structures were characterized by the same transmission distribution as a function of frequency. The symmetry of the layer substructure surrounded by a material with significantly lower acoustic impedance did not affect the transmission distribution. As the thickness of the defect increased, the transmission of the wave decreased. Peaks characteristic of selected structures were observed. The presence of a defect in the outer layers of the structure resulted in shifting the transmission peaks towards lower frequencies. Band gaps were observed.

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