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TUNING OF DEFORMATION MODELS BY RESIDUAL STRESS MEASUREMENTS

SKALOWANIE MODELU DEFORMACJI NA PODSTAWIE POMIARU NAPRĘŻEŃ WEWNĘTRZNYCH

Deformation models are generally used for prediction of mechanical tests, crystallographic textures and for interpretation of residual stress measurements. Existing models have very different levels of interaction between a grain and the surrounding material. If an isotropic interaction is assumed, the relation between micro- and macroscopic quantities contains only one interaction parameter (L). The simplest plastic deformation models, proposed by Sachs and Taylor, are based on the limiting assumptions of strain or stress homogeneity within the polycrystalline material; consequently L=0 in Sachs model and $L \to \infty$ in Taylor model. In more realistic approaches the L parameter has some finite value (and different from zero). In the early version of the self-consistent model (Berveiller and Zaoui), the L parameter was expressed as: $L=\alpha G$ (G being the shear modulus and α — the elasto-plastic accommodation parameter from the range [0, 1]). The aim of the present work is to find a real level of the interaction parameter for highly rolled steel. The estimation was done by comparing predicted and experimental data for residual stress and crystallographic textures.

Keywords: Deformation models, residual stress, second order stress, crystallographic texture, diffraction method, fitting procedure

Modele deformacji są na ogół używane do przewidywania rezultatów testów mechanicznych, tekstur krystalograficznych oraz do interpretacji pomiarów naprężeń wewnętrznych. Istniejące modele charakteryzują się bardzo różnym poziomem oddziaływania pomiędzy danym ziarnem i otaczającym materiałem. Jeśli założy się oddziaływanie izotropowe, to relacja pomiędzy wielkościami mikro- i makroskopowymi opisywana jest tylko jednym parametrem (L). Najprostsze modele deformacji, sformułowane przez Sachsa i Taylora, opierają się na granicznych założeniach jednorodności naprężenia lub deformacji w materiale; w konsekwencji otrzymujemy L=0 w modelu Sachsa oraz $L \to \infty$ w modelu Taylora. W bardziej realistycznym opisie deformacji elasto-plastycznej, parametr L posiada określoną, skończo-

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ną wartość (i różną od zera). We wczesnej wersji modelu samo- uzgodnionego (Berveiller, Zaoui), parametr L wyrażony został jako: L=αG (gdzie G jest modułem ścinania, zaś α jest współczynnikiem akomodacji elasto-plastycznej, przyjmującym wartości z przedziału [0, 1]). Celem obecnej pracy jest znalezienie realistycznego poziomu parametru oddziaływania dla walcowanej stali. Oszacowania tego dokonano przez porównanie przewidywanych i zmierzonych naprężeń szczątkowych oraz tekstur krystalograficznych.

1. Introduction

The essential point of any deformation model of polycristalline material is the interaction law, describing the relation between local and macroscopic stress and strain tensors (Fig. 1). There exist a wide varietv of deformation models. They describe

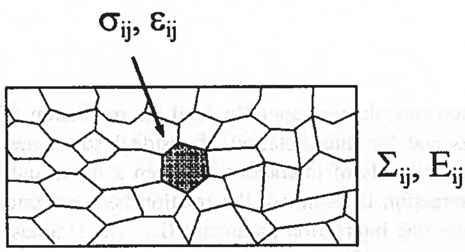


Fig. 1. Σ_{ij} stress tensor is applied to the sample and a grain is subjected to a local stress tensor σ_{ij}. Similarly, the sample deformation is E_{ij} but local grain deformation is ε_{ij}

macroscopic deformation of a material based on some micro-structural and crystallographic data and assumptions. The basic question of micro-macro transition can be formulated as follows:

$$(\Sigma_{ij}, E^{p}_{ij}, E^{e}_{ij}) \overset{?}{\Leftrightarrow} (\sigma_{ij}, \varepsilon^{p}_{ij}, \varepsilon^{e}_{ij}). \qquad (1)$$

The existing elasto-plastic models have very different interaction laws. However, in some approximation, a typical interacion law can be expressed as:

$$\dot{\sigma}_{ij} = \dot{\Sigma}_{ij} + L_{ijkl}(\dot{E}_{kl} - \dot{\varepsilon}_{kl}), \qquad (2)$$

where σ_{ij} and ε_{ij} are stress and strain of a grain, Σ_{ij} and E_{ij} are the same quantities for the sample, L_{ijkl} is the interaction tensor (the summation convention on the repeated lower index is used in this paper) and *dot* means the time derivative. If the isotropic interaction between a grain and its neighbours is assumed:

$$\dot{\sigma}_{ij} = \dot{\Sigma}_{ij} + L(\dot{E}_{ij} - \dot{\varepsilon}_{ij}). \qquad (3)$$

The interaction parameter L becomes a scalar one. The simplest plastic deformation models, proposed by S a c h s and T a y l o r [1, 2], are based on the limiting assumptions of strain or stress homogeneity within the polycrystalline material; consequently $L = 0$ in S a c h s model and $L \to \infty$ in T a y l o r model. A more realistic approach is based on the concept of self-consistent modelling of the elastic and plastic properties of inhomogeneous material. In the early version of the self-consistent elasto-plastic model proposed by B e r v e i l l e r and Z a o u i [4], the interaction parameter was expressed as: $L = \alpha G$, where G is shear modulus and α is the elasto-plastic accommodation parameter, taking values between 0 and 1. It is interesting to note that in a purely elastic model of K r ö n e r [3]: $\alpha \cong 1$. In general $\alpha < 1$ because an additional local glide appears in grain boundary regions and this reduces the inter-granular incompatibility. The aim of the present work was to find the real value of L (or of α), comparing predicted and measured residual stresses and textures.

2. Deformation model

The polycrystalline deformation model proposed by L e f f e r s [5] and developed by W i e r z b a n o w s k i [6] was used in the present work (LW model). A polycrystalline sample is represented in the model by 5000 grains with initial orientation distribution corresponding to the initial sample texture. The calculations are performed at two different scales, i.e.: at the macro-scale where the stress applied to the sample (Σ_{ij}) is defined and at a grain- scale where the behaviour of each crystallite under a local stress (σ_{ij}) is analysed. Only these slip systems <*uvw*> {*hkl*} are activated in grains for which the resolved shear stress $\tau_{<uvw>\{hkl\}}$ exceeds some critical value τ_c (Schmid law). The linear hardening of slip systems was used [7]:

$$\dot{\tau}_c^i = \sum_j H^{ij} \dot{\gamma}^j, \tag{4}$$

where: $\dot{\tau}_c^i$ is the rate of critical stress in the i-th system, $\dot{\gamma}^j$ is a rate of the plastic shear strain on the *j-th* system and H^{ij} is the work hardening matrix. In the present work the isotropic hardening matrix was used, i.e., $H^{ij} = h(for\ all\ i, j)$. The rolling process is described by the application of macroscopic load with $\Sigma_{11} = -\Sigma_{33}$, other components being zero. The amplitudes of both stress components are incrementally increasing during calculations in order to maintain slip systems activation in grains (τ_c^i are increasing). Calculations are performed since a preset final deformation is attained.

It has already been mentioned that the essential point of each model is the relation between macroscopic ($\dot{\Sigma}_{ij}$ and $\dot{E}_{ij}$) and grain ($\dot{\sigma}_{ij}$ and $\dot{\varepsilon}_{ij}$) quantities. In the present work, the isotropic approximation of grain-matrix interaction was used (Eq. 3). The model calculations furnish grain and sample deformations, mechanical tests predictions, crystallographic texture change, dislocation density and the state of residual stresses [6, 8-10].

3. Experimental method of residual stress determination

In the present work a general *multi-reflection method* of stress determination was applied (see e.g. [10]). Classical X-ray diffraction (Cr radiation) was used. The inter-planar distances $<d(\phi,\psi)>_{\{hkl\}}$ are measured for different hkl reflections and for various orientations of the scattering vector characterized by the ϕ and ψ angles (see Fig.2). If one deals *only* with the first order residual stress (σ^{I}_{ij}), the strain measured along the scattering vector (L_3 axis — Fig. 2) is:

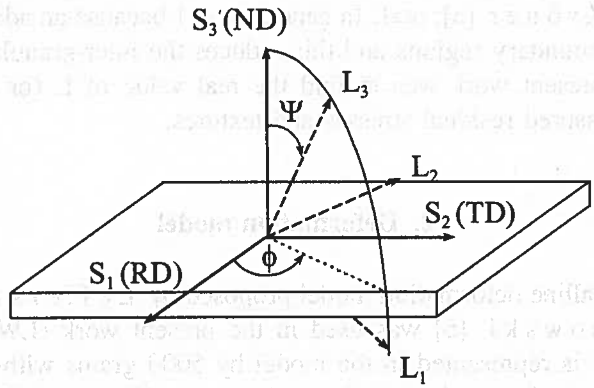


Fig. 2. Sample (S) and laboratory (L) systems used in the residual stress measurement by diffraction method (L_3 parallel to the scattering vector)

$$< \varepsilon'(\psi,\phi) >_{\{hkl\}} = F_{ij}(hkl,\psi,\phi,f(g))\,\sigma^{I}_{ij}, \tag{5}$$

where:

$$< \varepsilon'(\psi,\phi) >_{\{hkl\}} = \frac{< d(\psi,\phi) >_{\{hkl\}} - d^{o}_{\{hkl\}}}{d^{o}_{\{hkl\}}} \tag{6}$$

or in equivalent form:

$$< d(\phi,\psi) >_{\{hkl\}} = [F_{ij}(hkl,\psi,\phi,f(g))\,\sigma^{I}_{ij}]d^{o}_{\{hkl\}} + d^{o}_{\{hkl\}}, \tag{7}$$

where F_{ij}(hkl, ψ, ϕ, f(g)) are diffraction elastic constants (calculated using the self-consistent model and sample texture [11]); besides of ϕ and ψ angles and *hkl* reflection indices, they depend also on texture function $f(g)$ of the sample. In the multi-reflection method the equivalent lattice constants:

$$<a(\psi,\phi)>_{\{khl\}}=<d(\psi,\phi)>_{\{khl\}}\sqrt{h^2+k^2+l^2} \quad \text{and} \quad a^{o} = d^{o}_{\{khl\}}\sqrt{h^2+k^2+l^2} \tag{8}$$

are treated simultaneously in the fitting procedure [8-10]. The values of a^0 and σ^I_{ij} are found from Eqs. 7 and 8. The obtained results are statistically more representative in comparison with the single reflection method.

In the present work a plate of single phase ferritic steel, rolled to 95% reduction was examined (final thickness of 1 mm). The surface layer of about 200 μm was removed by electro-polishing. It was assumed that measurements were representative for the bulk volume of the material and that only σ^I_{33} macro-stress component was relaxed while the second order stresses were approximately unchanged. The $<a(\phi,\psi)>_{\{hkl\}}$ were determined for 211, 200 and 110 reflections and for various ϕ.

4. Determination of interaction parameter

The lattice strains $<\varepsilon'(\psi,\phi)>_{\{hkl\}}$ in plastically deformed material can be generally expressed as a superposition of strains induced by the macro- stresses and by the second order incompatibility stresses [8, 9]. Consequently Eq.5 will take more extended form:

$$< \varepsilon'(\phi,\psi) >_{\{hkl\}} = F_{ij}(hkl,\psi,\phi,f(g))\,\sigma^I_{ij} + < \gamma_{3m}\gamma_{3n}s_{mnij}\sigma^{II}_{ij} >_{\{hkl\}}, \qquad (9)$$

where s_{mnij} are single crystal elastic constants and σ^{II}_{ij} is the second order incompatibility stress defined for a grain (the latter quantities being expressed in **S** system, hence γ is the transformation matrix from **S** to **L** system — c.f. Fig. 2); $< \cdots >$ means the average over diffracting crystallites. The σ^{II}_{ij} incompatibility stress remains after unloading of the macro- stresses and is caused by different plastic deformations of grains. It can be approached from the model (Eq. 3) as:

$$\overline{\dot{\sigma}^{II}_{ij}} \cong L(\dot{E}_{ij} - \dot{\varepsilon}_{ij}), \qquad (10)$$

where $\overline{\sigma^{II}_{ij}}$ is the *model predicted* second order incompatibility stress. We suppose, that anisotropy of incompatibility stresses is well predicted by the model. However, the absolute stress level depends on complex hardening processes which are not completely described by the simple linear hardening law used in the model (Eq. 4). This is the reason why the predicted $\overline{\sigma^{II}_{ij}}$ stress tensor is rescaled using some factor (q) :

$$\sigma^{II}_{ij}(g) = q\,\overline{\sigma^{II}_{ij}}(g). \qquad (11)$$

Finally, the strain measured in diffraction experiment is expressed as:

$$< \varepsilon'(\phi,\psi) >_{\{hkl\}} = F_{ij}(hkl,\psi,\phi,f(g))\,\sigma^I_{ij} + q < \gamma_{3m}\gamma_{3n}\, s_{mnij}\overline{\sigma^{II}_{ij}} >_{\{hkl\}}. \qquad (12)$$

Or taking into account Eqs 6 and 8:

$$< a(\psi,\phi) >_{\{hkl\}} = [F_{ij}(hkl,\psi,\phi,f(g))\,\sigma^I_{ij} + q < \gamma_{3m}\gamma_{3n}s_{mnij}\overline{\sigma^{II}_{ij}} >]a^o + a^o \qquad (13)$$

Comparing measured and predicted $<a(\psi,\phi)>_{\{hkl\}}$ vs $\sin^2\psi$ relations, σ^I_{ij}, a_o and q can be determined as fitting parameters. The additional term $q <\gamma_{3m}\gamma_{3n}s_{mnij}\overline{\sigma^{II}_{ij}}>$, characterizing non-linearities of the $\sin^2\psi$ plot, improves considerably the agreement between calculated and experimental data — Fig. 3. The best fit was obtained for the

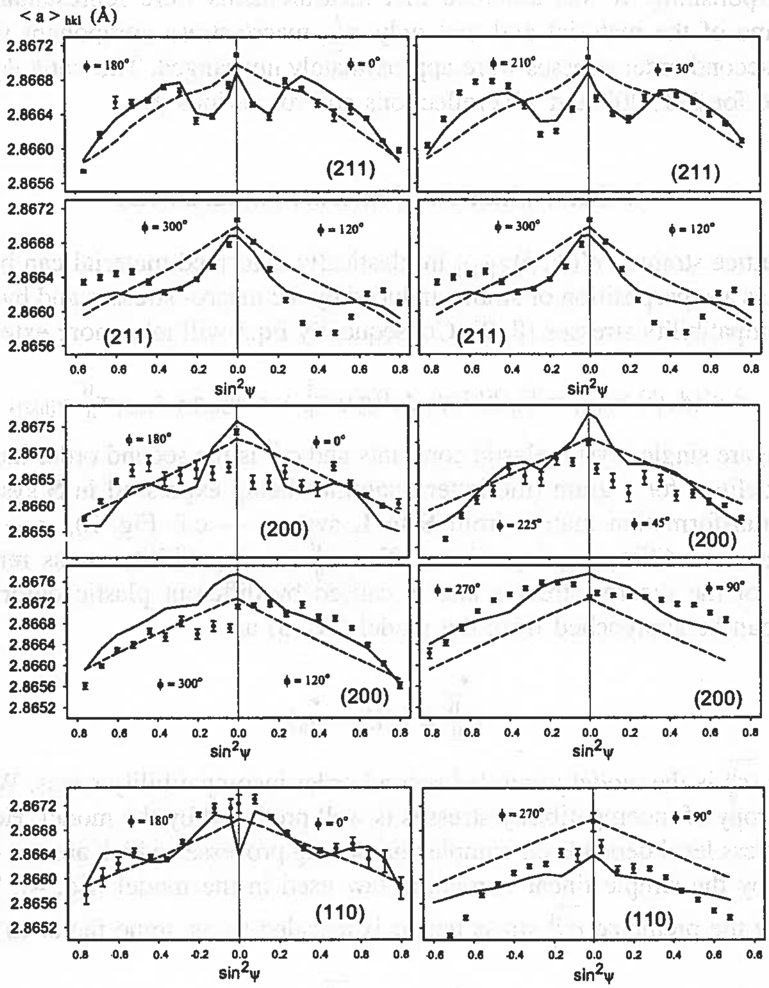
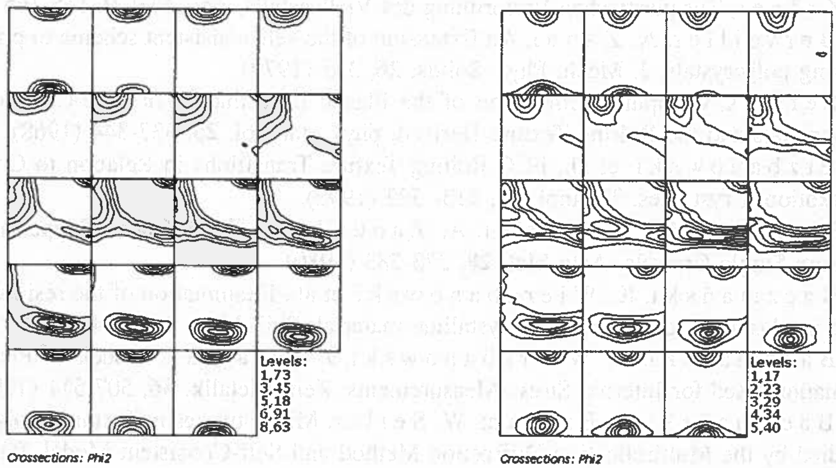


Fig. 3. Measured lattice parameters (points) and theoretical results of fitting (continuous lines for q ≠ 0 and lines for q = 0) for cold rolled steel. The experimental data for various *hkl* reflections were simultaneously used in the fitting procedure. LW model (*L* = 800 MPa) was used

value L=800 MPa ($\alpha \cong 0.01$) of the model interaction parameter. Also the predicted and experimental textures are in good agreement in this case (Fig. 4).

The following non-zero components of the first order stress were obtained from our measurements: σ^I_{11} = -60.4 MPa, σ^I_{22} = -72.6 MPa. On the other hand, the estimated



a **b**

Fig. 4. a) Predicted (LW model, L=800 MPa), and b) measured rolling texture of low carbon steel; rolling reduction 60%

Von Mises measure of the second order incompatibility stresses σ_{ij}^{II} was 55 MPa (q=1.27 was found). Hence, the second order incompatibility stresses have comparable order of magnitude as the first order stresses and they have important influence on local behaviour of grains.

5. Conclusions

The proposed method enables evaluation of the interaction level in deformation models. It is based on the comparison of measured and predicted $\langle a \rangle_{hkl}$ vs $\sin^2\psi$ relations. The best adjustment for the rolled steel samples leads to the value of L = 800 MPa ($\alpha \cong 0.01$) which corresponds also to the best texture prediction. This means that mutual grain-grain interactions are about 100 times weaker compared with purely elastic relation ($\alpha \cong 1$). This "softening" of interaction can be explained by some additional local glide in the region of grain boundaries. It reduces the grain-matrix plastic incompatibility and hence the values of residual second-order stresses; they have comparable level as the first order stresses.

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