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ENERGY-BASED YIELD CONDITION FOR ORTHOTROPIC MATERIALS EXHIBITING ASYMMETRY OF ELASTIC RANGE

The aim of the paper is to formulate physically well founded yield condition for initially anisotropic solids revealing the asymmetry of elastic range. The initial anisotropy occurs in material primarily due to thermo-mechanical pre-processing and plastic deformation during the manufacturing processes. Therefore, materials in the "as-received" state become usually anisotropic. After short account of the known limit criteria for anisotropic solids and discussion of mathematical preliminaries the energy-based criterion for orthotropic materials was formulated and confronted with experimental data and numerical predictions of other theories. Finally, possible simplifications are discussed and certain model of isotropic material with yield condition accounting for a correction of shear strength due to initial anisotropy is presented. The experimental verification is provided and the comparison with existing approach based on the transformed-tensor method is discussed.
Keywords: Energy-based yield condition, Orthotropic solids, Initial anisotropy, Strength differential effect, Numerical simulation

1. Introduction

Recent achievements of materials science are related with search for better understanding of deformation mechanisms and failure processes in newly designed and manufactured materials. The analysis of thermomechanical processes related with many industrial and laboratory applications should account for some unconventional mechanical and functional properties of investigated solids. Among the uncommon properties of materials one should distinguish in particular, the tailored anisotropy of mechanical properties by multiscale controlling the structure on the macroscopic, microscopic or nanoscale level. Also asymmetry of elastic range produced by multiscale processes as for instance by multilevel hierarchy of shear banding, [27-29] can be accounted for. The latter transpires as the difference of tensile and compression strength – the so called strength differential effect (SDE), pressure sensitivity of yield limit and the dependency on the third invariant of stress deviator described by Lode angle on the octahedral plane as well as anisotropy of yield states. One of the goals of modern mechanics of solids is to provide such a mathematical description, which enables to account for these features. The aim of the paper is to formulate physically well founded yield condition for orthotropic solids revealing the asymmetry of elastic range. The materials with initial anisotropy in the "as-received" state are also considered.

The advantage of energy-based measure of material effort lies in the multiscale character of energy concept. This means that the measure of material effort has well defined physical interpretation in each scale of considerations, starting with quantum mechanical sense of atomic bonds through phenomenological potentials in Molecular Dynamics simulations of interatomic interactions till modelling of cohesion on the micro- and macroscopic scale. In this way there is the possibility to estimate the critical values of material effort in the cases when the specimens of investigated novel material are too small for reliable experimental determination, cf e.g. the investigations related with deformation and limit states of glassy metals [14] and [29].

The novelty of the proposed approach is based on the hypothesis that certain components of elastic energy density are involved in formulating the measure of material effort. The notion of material effort can be defined as the state of material point of the body related with the change of strength of chemical bonds with respect to the natural state. The discussion of physical

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basis of this concept is contained in [29]. This definition is based on the earlier studies of the relation of microscopic observations to modelling of viscoplastic flow and failure of solids by [26] as well as on the applications of Molecular Dynamics for the analysis of the strength of chemical bonds determining, as an example, the strength of metal-metal oxide interfaces studied in [20] and [19]. Historical account of the idea to use the elastic energy density as the material effort measure is given in [29].

A definite influence of pressure on the yield stress was proved by experiments, cf. e.g. [33,30] and [39]. If the distinct difference of these values is observed, the asymmetry of elastic range appears. The origins of SDE, which is deeply rooted in the electronic basis of material structure can be found in the calculations of UBER function (Universal Binding Energy Relation), discussed for instance for Cu by [18] and also in [29].

The complete solution for the case of isotropic body as well as for a kind of anisotropy with isotropic volumetric changes was provided by [2,3], cf also [38] and [1] where some new applications of Burzyński's criterion in numerical solutions of thermo-plasticity problems related with the strength of the turbine blades was discussed.

Mises in his fundamental paper, [15], has presented the thorough study of limit criteria for anisotropic solids. An array of works following the original ideas of Mises was discussed in more detail in [25], where the yield criteria by [8,9] and [37], as well as, [4] were considered.

The formulation of energy-based hypothesis of material effort for anisotropic solids revealing SDE was attempted by [24,25,35] and [34]. The study is based on the concept of energy-orthogonal decompositions of stress state introduced by [31,32] and [23]. The concept of stress state dependent functions describing the influence of certain stress modes on the total measure of material effort was firstly presented by [2,3]. General formulation of a new limit criterion as well as its specification for certain elastic symmetries was given in [25] and general methodology of acquiring necessary data for the criterion specification was presented. The general formulation was specified for orthotropic materials and formulated both for two-dimensional and three-dimensional cases by [34]. Explicit formulae based on simple strength tests were derived for parameters of criterion in the plane case. However, according to authors opinion, the elaboration of complete methodology of theoretical specification of limit criteria for assumed material symmetries and its experimental identification requires still much work and it is worthwhile further research.

Independently, the discussed requirement of accounting for SDE in orthotropic materials was fulfilled in mathematically precise and comprehensive way by [22]. The authors presented a general definition of an explicit orthotropic yield criterion together with a general method for defining implicit orthotropic yield functions. The latter formulation is based on the transformed-tensor method. The principal advantage lies in the possibility of adjusting an arbitrary isotropic yield criterion to the behaviour of an anisotropic material. As example, the adjustment to the [8,9] and [37] criteria was performed. These particular cases illustrated the more general methodology for obtaining the desired function identification for any other well-known criterion or set of experimental data. The analysis of the, considered in [22], models of isotropic limit criteria leads to the novel observation that application of the mentioned above energy-based approach by [2,3] provides the similar formulation of limit conditions with clear physical interpretation. Therefore. the goal of the presented study is the original derivation of possible simple formulation of limit criteria for materials exhibiting orthotropy or initial anisotropy and strength differential effect.

In Section 2 it is presented the mathematical foundations of energy-based limit criterion for orthotropic materials with an account for the asymmetry of the elastic range. The initial anisotropy effects are discussed and the corresponding multi-surface Burzyński criterion is determined in the principal stress space. The smooth approximation of the multi-surface criterion for initially anisotropic materials revealing SDE is derived. In Section 3 the comparison with experimental data for Al2008-T4 and simulation results for other known criteria published in the literature is provided. The comparison of own experimental results obtained for E335 steel with the smooth approximation of the multi-surface criterion without SD effect, is presented in Fig. 5.

2. Mathematical foundations

2.1. Introduction

As it was mentioned above, strictly quadratic energy-based limit criteria cannot account for the asymmetry of the elastic range. This was one of the reasons for which [2,3] extended the aforementioned limit criterion of [10,11] in order to account for the SDE. The author considered only a special class of linear elastic materials for which the decomposition of the elastic energy density into the sum of energy of volume change Φ_v and energy of distortion Φ_f is possible also in some cases of anisotropy:

$$\Phi = \frac{1}{2}\boldsymbol{A}_\sigma \cdot \boldsymbol{A}_\varepsilon + \frac{1}{2}\boldsymbol{D}_\sigma \cdot \boldsymbol{D}_\varepsilon \qquad (1)$$

where $\boldsymbol{A}_\sigma$, $\boldsymbol{A}_\varepsilon$ and $\boldsymbol{D}_\sigma$, $\boldsymbol{D}_\varepsilon$ denote the spherical and deviatoric part of stress and strain tensors respectively. Such a class of linearly elastic anisotropic materials is called volumetrically isotropic. It is worthy to note that the problem of volumetric – distortional decomposition considered originally by Burzyński in 1928 was undertaken more recently by [36] and [5]. Also in [12] the concept of volumetrically isotropic solids with orthotropic symmetry was studied and the energy-based interpretation of the yield criterion for orthotropic materials by [8] was discussed.

2.2. Energy-based criterion for orthotropic materials

Consider the linear elastic material having one of eight possible symmetry classes – orthotropy, ([13]). For such a case

[2,3] derived, according to Eq. (1) the following volumetric – distortional decomposition of elastic energy density that can be expressed in the orthotropy axes (x, y, z) in the following form:

$$\Phi = \frac{1}{2}B^*\left(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}\right)^2 + \frac{1}{3}\left[\begin{array}{l} N^*\left(\sigma_{xx}-\sigma_{yy}\right)^2 + L^*\left(\sigma_{yy}-\sigma_{zz}\right)^2 \\ +M^*\left(\sigma_{zz}-\sigma_{xx}\right)^2 \end{array}\right] + 2\left(P^*\sigma_{xy}^2 + Q^*\sigma_{xz}^2 + R^*\sigma_{yz}^2\right) \tag{2}$$

where B^* denotes the compliance modulus of volumetric change, while N^*, L^*, M^* are the compliance moduli of distortional changes in three perpendicular planes. This original derivation made a basis for mathematical formulation of a new yield condition for anisotropic but volumetrically isotropic materials with asymmetry of elastic range (accounting for SDE).

$$\Phi_f + \eta(p)\Phi_v = K$$
$$\eta = \omega + \frac{\delta}{3p}, \quad p = \frac{\sigma_{xx}+\sigma_{yy}+\sigma_{zz}}{3} \tag{3}$$

Let's introduce the following notation:

- σ_{xx}^T – tensile yield stress for direction x,
- σ_{yy}^T – tensile yield stress for direction y,
- σ_{zz}^T – tensile yield stress for direction z.
- σ_h^C – tri-axial compression yield stress.

Substituting the following stress states

- $(\sigma_{xx}=\sigma_{xx}^T, \sigma_{yy}=0, \sigma_{zz}=0)$
- $(\sigma_{xx}=0, \sigma_{yy}=\sigma_{yy}^T, \sigma_{zz}=0)$
- $(\sigma_{xx}=0, \sigma_{yy}=0, \sigma_{zz}=\sigma_{zz}^T)$
- $(\sigma_{xx}=-\sigma_h^C, \sigma_{yy}=-\sigma_h^C, \sigma_{zz}=-\sigma_h^C)$

to Eq. (3) gives the system of the algebraic equations

$$\frac{1}{2}B^*(\sigma_{xx}^T)^2(\frac{\delta}{\sigma_{xx}^T}+\omega)-K + \frac{1}{3}L^*(\sigma_{xx}^T)^2 + \frac{1}{3}N^*(\sigma_{xx}^T)^2 = 0 \tag{4}$$

$$\frac{1}{2}B^*(\sigma_{yy}^T)^2(\frac{\delta}{\sigma_{yy}^T}+\omega)-K + \frac{1}{3}L^*(\sigma_{yy}^T)^2 + \frac{1}{3}M^*(\sigma_{yy}^T)^2 = 0 \tag{5}$$

$$\frac{1}{2}B^*(\sigma_{zz}^T)^2(\frac{\delta}{\sigma_{zz}^T}+\omega)-K + \frac{1}{3}M^*(\sigma_{zz}^T)^2 + \frac{1}{3}N^*(\sigma_{zz}^T)^2 = 0 \tag{6}$$

$$\frac{9}{2}B^*(\sigma_h^C)^2\left(\frac{-\delta}{3\sigma_h^C}+\omega\right)-K = 0 \tag{7}$$

Solving the system of equations (4-7) for M^*, N^*, L^* and B^* gives

$$B^* = \frac{2}{3}\frac{K}{\sigma_h^C(\delta-3\omega\sigma_h^C)} \tag{8}$$

$$M^* = \frac{3}{4}B^*\delta\left(\frac{1}{-\sigma_{zz}^T}-\frac{1}{\sigma_{yy}^T}+\frac{1}{\sigma_{xx}^T}\right)-\frac{3}{2}B^*\omega + \frac{3}{2}K/(\sigma_{zz}^T)^2 + \frac{3}{2}K/(\sigma_{yy}^T)^2 - \frac{3}{2}K/(\sigma_{xx}^T)^2 \tag{9}$$

$$N^* = \frac{3}{4}B^*\delta\left(\frac{1}{-\sigma_{zz}^T}+\frac{1}{\sigma_{yy}^T}-\frac{1}{\sigma_{xx}^T}\right)-\frac{3}{4}B^*\omega + \frac{3}{2}K/(\sigma_{zz}^T)^2 - \frac{3}{2}K/(\sigma_{yy}^T)^2 + \frac{3}{2}K/(\sigma_{xx}^T)^2 \tag{10}$$

$$L^* = \frac{3}{4}B^*\delta\left(\frac{1}{\sigma_{zz}^T}-\frac{1}{\sigma_{yy}^T}-\frac{1}{\sigma_{xx}^T}\right)-\frac{3}{2}B^*\omega - \frac{3}{2}K/(\sigma_{zz}^T)^2 + \frac{3}{2}K/(\sigma_{yy}^T)^2 + \frac{3}{2}K/(\sigma_{xx}^T)^2 \tag{11}$$

Now the Burzyński criterion is defined by seven unknown parameters (σ_{xx}^T, σ_{yy}^T, σ_{zz}^T, σ_h^C, K, δ, ω). The parameter K can be assumed to be equal 1 because it scales only the values of the other parameters. Additionally, three experiments performing shears in the orthotropy planes (x, y), (x, z) and (y, z) should be used to specify the constants P^*, Q^* and R^*. As it is underlined by [2,3]), the determined in the relations Eqs. (8-11) parameters correspond to the elastic state limit, which in particular can correspond to the state of plastic yield. In such a case these parameters describe the limit state of the solid body and not elastic constants.

2.3. Description of the initial anisotropy effects in application to metals

Initial anisotropy occurs in the material that is in the "as-received" state due to different pre-processing methods of manufacturing. Our objective is to propose certain simplified account for the aforementioned initial anisotropy with use of the model of isotropic material as regards elastic and plastic yield properties with certain correction of the yield shear strength. In materials with a slight initial anisotropy which is produced for instance by texture induced in manufacturing process, the main effect which is observed is deviation of shear strength with respect to the predicted one from the Huber-Mises yield condition. Such an effect can be described in a relatively simple way within the framework of the model for isotropic material with use of certain shear strength correction factor – λ the multi-surface Burzyński criterion is determined in the space of the principal stresses. Burzyński derives the factor λ as a constant resulting from certain relations between the elastic moduli for anisotropic solid. However, to reduce the number of constants in the yield criterion Burzyński replaces the description of considered anisotropic material with a model of isotropic solid. The whole effect of anisotropy is contained in the additional correction factor λ, which may be also explicitly calculated using the results of strength tests. Substituting the following relationship considered by Burzyński [3], p. 206.

$$\frac{1-2\nu^*}{1+\nu^*}=\frac{3}{2}\frac{B^*}{L^*}\frac{M^*}{N^*}\omega,$$
$$\frac{3(\sigma_Y^C-\sigma_Y^T)}{1+\nu^*}=\frac{3}{2}\frac{B^*}{L^*}\frac{M^*}{N^*}\delta,$$
$$\frac{3\sigma_Y^C\sigma_Y^T}{1+\nu^*}=\frac{3KM^*}{L^*N^*} \tag{12}$$

to Eq. (2) we get

$$\frac{1+\nu^*}{3}\left(\sigma_f^*\right)^2+3\left(1-2\nu^*\right)p^2$$
$$+3\left(\sigma_Y^C-\sigma_Y^T\right)p-\sigma_Y^C\sigma_Y^T=0 \tag{13}$$

where

$$\left(\sigma_f^*\right)^2=\frac{M^*}{N^*}(\sigma_2-\sigma_3)^2$$
$$+2\lambda(\sigma_3-\sigma_1)^2+\frac{M^*}{L^*}(\sigma_1-\sigma_2)^2 \tag{14}$$

Assuming that

$$\frac{M^*}{L^*}=\frac{M^*}{N^*}=2(1-\lambda) \tag{15}$$

the final formula is given by

$$(1-\lambda)(\sigma_2-\sigma_3)^2+\lambda(\sigma_3-\sigma_1)^2+(1-\lambda)(\sigma_1-\sigma_2)^2$$
$$+(\sigma_Y^C-\sigma_Y^T)(\sigma_1+\sigma_2+\sigma_3)=\sigma_Y^C\sigma \tag{16}$$

where

$$\lambda=\frac{\sigma_Y^C\sigma_Y^T}{2\tau_Y^2}-1 \tag{17}$$

The relation (15) was derived in Burzyński [2] and [3] considering two special cases of the formula (14) for uniaxial tension and uniaxial compression. The λ parameter takes values in the range $0<\lambda<1$. In particular case if $\lambda = 0.5$ then there is obtained the paraboloid Burzyński criterion for isotropic materials. And if also $\sigma_Y^C=\sigma_Y^T=\sigma_Y$ then the Burzyński criterion changes to the Huber-Mises criterion.

Based on the previous parametric study, it is clear that some singularities are observed which may induce some problems during computing, Fig. 1.

2.4. Smooth approximation method of criterion for initially anisotropic materials

If the criterion is to be implemented in FEM codes, the corners visible in Fig. 1 should be avoided. The new limit curve is a result of approximation with use of fitting parameters A_i, B_i, C_i – Eqs. (18), which should fulfil conditions covered by Eqs. (19-22), see [21].

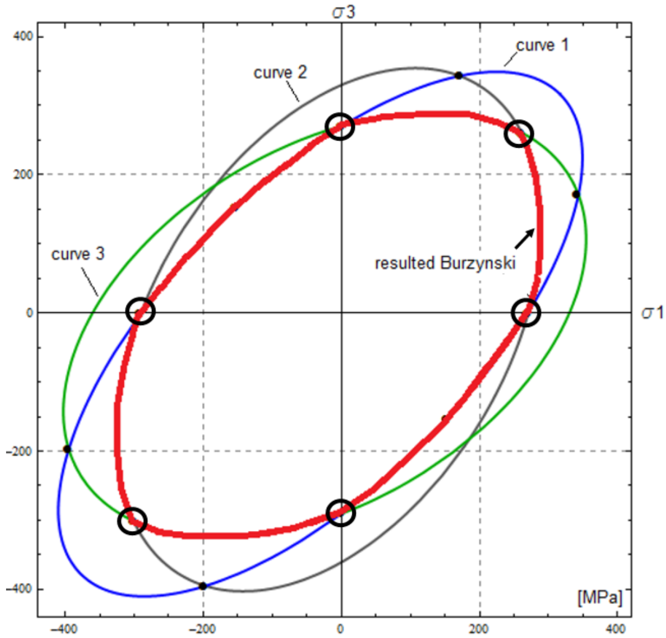

Fig. 1. The cross-section of the Burzyński multi-surface criterion in the plane of principal stresses $\sigma_3 - \sigma_1$ with black circle marks of the singularity points

$$\Phi_1(\sigma_1,\sigma_3)=\sigma_1^2-A_1\sigma_1\sigma_3+A_2\sigma_3^2$$
$$+A_3(\sigma_Y^C-\sigma_Y^T)\sigma_1+A_4(\sigma_Y^C-\sigma_Y^T)\sigma_3-\sigma_Y^C\sigma_Y^T$$
$$\Phi_1(\sigma_1,\sigma_3)=\sigma_1^2+\sigma_3^2-B_1\sigma_1\sigma_3$$
$$+B_2(\sigma_Y^C-\sigma_Y^T)\sigma_1+B_3(\sigma_Y^C-\sigma_Y^T)\sigma_3-\sigma_Y^C\sigma_Y^T \tag{18}$$
$$\Phi_1(\sigma_1,\sigma_3)=C_1\sigma_1^2-C_1\sigma_1\sigma_3+\sigma_3^2$$
$$+C_3(\sigma_Y^C-\sigma_Y^T)\sigma_1+C_4(\sigma_Y^C-\sigma_Y^T)\sigma_3-\sigma_Y^C\sigma_Y^T$$

To obtain parameters A_i, B_i, C_i the following relations must be solved, Eqs. (19-22). The conditions for smoothness of a function of the Burzyński criterion are presented as follows:
for uniaxial tension:

$$\sigma_1=\sigma_Y^T,\ \sigma_3=0$$
$$\Phi_1(\sigma_1,\sigma_3)=0,\ \Phi_2(\sigma_1,\sigma_3)=0 \tag{19}$$

for uniaxial compression:

$$\sigma_1=-\sigma_Y^C,\ \sigma_3=0$$
$$\Phi_1(\sigma_1,\sigma_3)=0,\ \Phi_2(\sigma_1,\sigma_3)=0 \tag{20}$$

$$\sigma_1=0,\ \sigma_3=-\sigma_Y^C$$
$$\Phi_2(\sigma_1,\sigma_3)=0,\ \Phi_3(\sigma_1,\sigma_3)=0 \tag{21}$$

for biaxial compression:

$$\sigma_1=-\sigma_Y^{CC},\ \sigma_3=-\sigma_Y^{CC}$$
$$\Phi_1(\sigma_1,\sigma_3)=0,\Phi_3(\sigma_1,\sigma_3)=0 \tag{22}$$

In Fig. 2 there are indicated points in the cross-section $\sigma_2 = 0$ in which the subsequent surfaces of the multi-surface Burzyński criterion are valid. The equality of first derivatives is necessary in the points (A), (B), (C) plotted in Fig. 2:

- points (A):

$$\frac{\partial\Phi_1(\sigma_1,\sigma_3)}{\partial\sigma_1} = \frac{\partial\Phi_2(\sigma_1,\sigma_3)}{\partial\sigma_1},$$
$$\frac{\partial\Phi_1(\sigma_1,\sigma_3)}{\partial\sigma_2} = \frac{\partial\Phi_2(\sigma_1,\sigma_3)}{\partial\sigma_2}$$

- points (B):

$$\frac{\partial\Phi_1(\sigma_1,\sigma_3)}{\partial\sigma_1} = \frac{\partial\Phi_3(\sigma_1,\sigma_3)}{\partial\sigma_1},$$
$$\frac{\partial\Phi_1(\sigma_1,\sigma_3)}{\partial\sigma_2} = \frac{\partial\Phi_3(\sigma_1,\sigma_3)}{\partial\sigma_2} \quad (23)$$

- points (C):

$$\frac{\partial\Phi_3(\sigma_1,\sigma_3)}{\partial\sigma_1} = \frac{\partial\Phi_2(\sigma_1,\sigma_3)}{\partial\sigma_1},$$
$$\frac{\partial\Phi_3(\sigma_1,\sigma_3)}{\partial\sigma_2} = \frac{\partial\Phi_2(\sigma_1,\sigma_3)}{\partial\sigma_2}$$

Consequently, after solving the above equations the final formulation of the Burzyński criterion is given by

$$\sigma_1^2 - R_B\sigma_1\sigma_3 + \sigma_3^2$$
$$+(\sigma_Y^C - \sigma_Y^T)(\sigma_1 + \sigma_3) - \sigma_Y^C\sigma_Y^T = 0 \quad (24)$$

where:

$$R_B = 2 - \frac{\sigma_Y^C\sigma_Y^T + 2(\sigma_Y^C - \sigma_T^C)\sigma_Y^{CC}}{(\sigma_Y^{CC})^2}$$

and σ_Y^{CC} is a yield stress in biaxial compression.

The comparison between the component curves the multisurface criterion and the final Burzyński criterion for initially anisotropic materials, Eq. (24), is presented in Fig. 3. The final yield surface obtained with use of smooth-out criterion with coefficient R_B is an elliptic paraboloid.

This model has been used to present the material description mainly in terms of yield surface for different materials as described in Section 3.

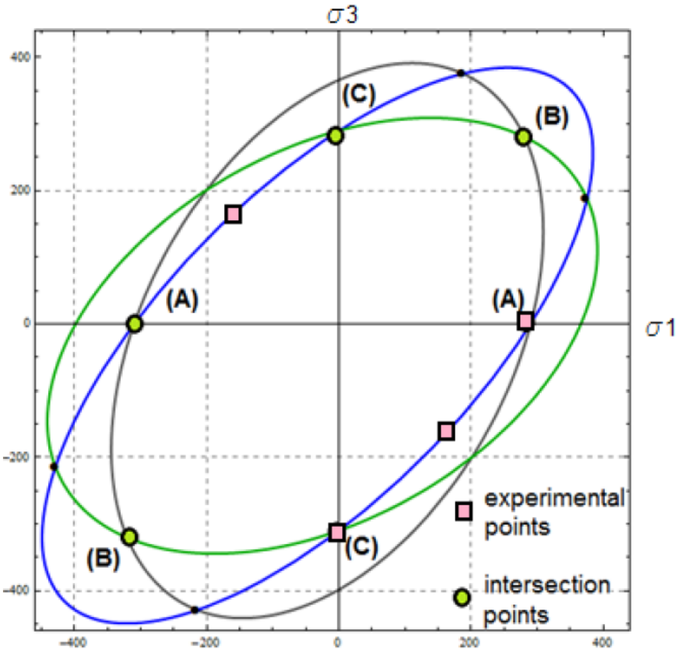

Fig. 2. Experimental points and points of intersection which are used to find the final formulation of the Burzyński criterion, Eq. (18). The experimental data are for OFHC copper σ_Y^C = 294 MPa point (C) obtained in uniaxial compression test, σ_Y^T = 272 MPa point (A) obtained in uniaxial tension test and τ_Y = 156 MPa obtained in pure shear test point the midpoint of the section (A-C), the strain rate 0.001 s^{-1}

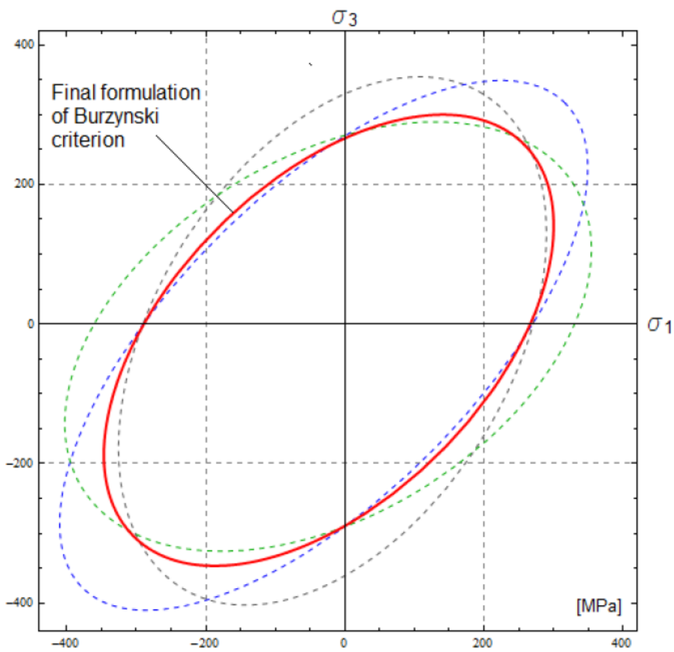

Fig. 3. Approximation of the inter-section of the Burzyński multi-surface criterion with a final formulation, Eq. (24)

3. Verification of proposed theory

3.1. Comparison with experimental data for Al2008-T4

In the papers of [16,17] the experimental data for the commercial Al2008-T4 aluminium alloy sheet of the yield stress directionality measured for uniaxial tensile and uniaxial compression tests were applied for the verification of considered yield criteria, Fig. 4. In the same Figure the proposed yield condition in Section 2.2 is included. It can be observed good correlation of the proposed criterion with the experimental data shown in Table 1, besides the range of biaxial compression. However, it can be noted that similar discrepancy occurs in the case of Hill 48 and Yld 2000-2d yield conditions. The unknown parameters of the proposed criterion were calculated with use of the Levenberg-Marquardt algorithm (LMA) which solves the nonlinear least squares problems. During the optimization process the algorithm fits the function with free parameters to a set of experimental data points by minimizing the sum of the squares of the errors between the experimental data and the function.

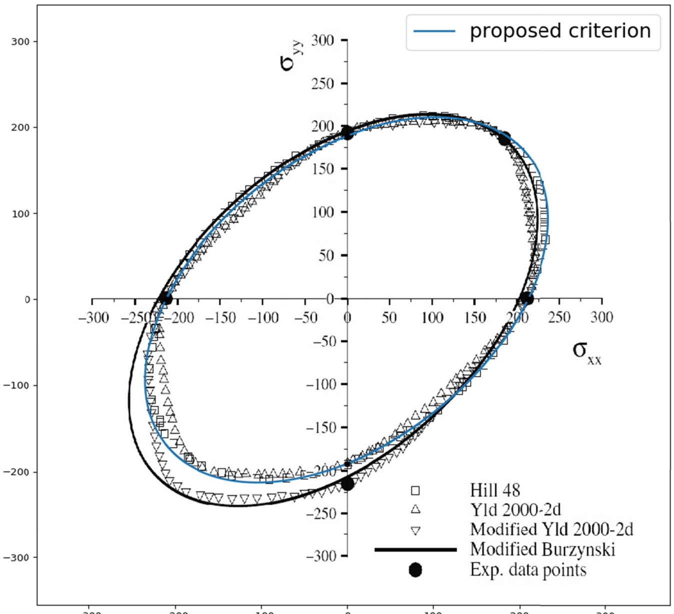

Fig. 4. Comparison of the proposed criterion with the experimental data and simulation results taken from [16,17]

The parameters used to describe experimental observations are reported in Table 1.

TABLE 1

Yield strength (MPa) of Al2008-T4 in tension and compression

σ_0^T	σ_{90}^T	σ_0^C	σ_{90}^C	σ_B^T
211.67	191.56	213.79	214.64	185.00

A good agreement is observed between the Burzyński model and experiments. To complete this approach a steel E335 has been used showing another shape of yield surface.

3.2. Comparison with own experimental results for E335 steel

The material considered in this part is a steel E355 with a BCC microstruture in comparison with the previous one which is an aluminium alloy with a FCC microstructure. In [6] the results of quasi-static investigation of thin-walled tubes of E335 steel subjected to complex states of stress are discussed and the yield surface is determined. In Fig. 5 the limit yield curves obtained according to the smooth approximation of Burzyński yield condition (red line) and the classical Huber-Mises criterion (dashed blue line), confronted with the experimental data are presented. It can be observed that there is a remarkable difference between the both criteria. The reason is that there is a difference between yield point in shear obtained experimentally (325 MPa) and calculated due to theoretical relation $\tau_Y = \sigma_Y / \sqrt{3}$. The difference is almost 20%, because the theoretical value equals to 260 MPa.

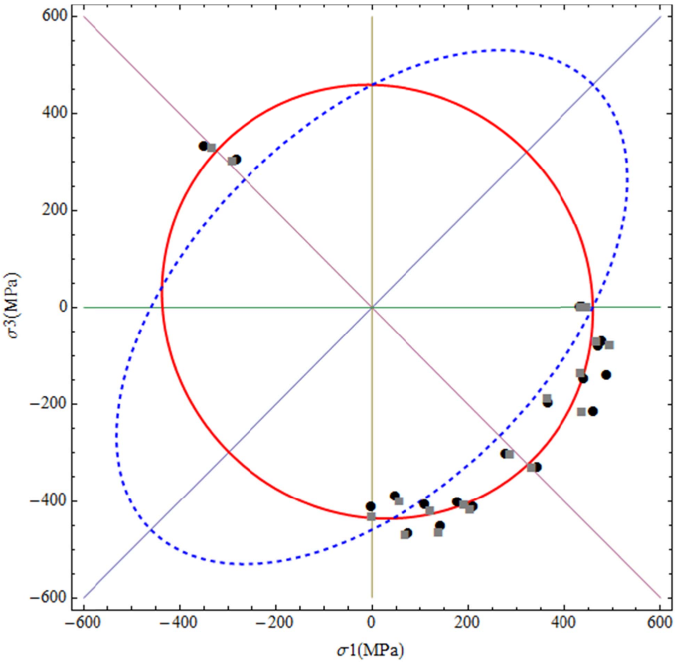

Fig. 5. The limit curves describing the experimental data for E335 steel, the red line – Burzyński yield condition, the dashed blue line Huber-Mises criterion [6]

The description of other materials is reported in [6,7] including metals and different polymers. In each case similarly as in Fig. 4 and Fig. 5 the smooth approximation of Burzyński criterion was applied to describe the yield surface and to verify with experimental observations.

4. Conclusions

Recently, the most popular in the studies of elastic-plastic deformation processes in anisotropic solid, particularly in orthotropic one, is the approach based on the transformed-tensor method summarized by [22]. Such an approach was applied also in [16,17] where the [2,3] hypothesis states that the elastic energy density of distortion and certain part of the energy density of volumetric changes are involved in formulating the measure of material effort. In the paper the authors attempted to show that the application of the mentioned above energy-based approach by [2,3] provides the similar formulation of limit conditions with clear physical interpretation. As a result, the simple formulation of limit criteria for materials exhibiting initial anisotropy and strength differential effect has been obtained. The study is confined to the materials that can be described, at least in a simplified way, by the class of volumetrically isotropic solids. In such a case the formulation of the new yield condition for orthotropic solids discussed in the Section 2.2 is possible.

Verification with the experimental data discussed in the Section 3.2 and comparison with the existing yield criteria show that the proposed yield condition is a satisfactory approximation, which can be used in numerical simulations of industrial deformation processes. Also the next step of approximation of the description of materials with initial anisotropy with use of the model of isotropic solid is considered. The effects of initial anisotropy are merely concentrated in a certain correction of the shear strength limit vis-à-vis the Huber-Mises criterion, as discussed in the Section 2.4 as well as in [7] and [6], finds the confirmation with experimental results and can be applied in the simple analysis of deformation processes, cf. [21].

It is worth mentioning that the other original approach presented in [40,41] leads to the analysis of anisotropic yield condition with application of fractional calculus. It might open a possibility of the energy-based Burzyński limit criterion as a basis for fractional formulation of viscoplasticity and plasticity models.

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