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AN EFFECT OF THE YIELD CRITERION ON THE POSITION OF THE BI-MATERIAL INTERFACE IN PLANAR FLOW OF THREE-LAYER MATERIAL THROUGH A WEDGE-SHAPE CHANNEL

WPŁYW KRYTERIUM UPLASTYCZNIENIA NA POŁOŻENIE GRANICY ROZDZIAŁU FAZ BIMATERIAŁU TRÓJWARSTWOWEGO W PŁASKIM PŁYNIĘCIU PRZEZ MATRYCĘ KLINOWĄ

The main objective of the paper is to show that the yield criteria chosen can have a significant effect on engineering applications of plane-strain rigid/plastic solutions for piece-wise homogeneous materials. The starting point of the concept is that experimental data are usually available for the tensile yield stress whereas the mathematical formulation requires the shear yield stress. Due to this fact, the interpretation of solutions for engineering applications depends on the yield criterion chosen for each material of multi- material composition. The paper deals with the problem of planar flow of three-layer material through a convergent wedge-shaped channel. Results of this research may be applied in the modeling of plastic deformation processes such as extrusion and drawing, to improve the accuracy of theoretical prediction of simultaneous plastic flow of materials with different mechanical properties.

Keywords: yield criterion, bi-material interface, planar flow, three-layer material, wedge-shape channel

W pracy pokazano, że wybór kryterium uplastycznienia w modelu ciała sztywno idealnie plastycznego, często używanym w zastosowaniach inżynierskich, może mieć bardzo istotny wpływ na wynik końcowy modelowania jeśli chodzi o materiały złożone. Z badań eksperymentalnych zwykle znane jest naprężenie uplastyczniające wyznaczone w teście na rozciąganie podczas gdy w modelowaniu matematycznym niezbędna jest wiedza o naprężeniu uplastyczniającym w ścinaniu, które się określa jednoznacznie z wyborem kryterium uplastycznienia (np. Treski czy Hubera-Misesa). Jeśli w przypadku procesów z monomateriałami wybór taki ma zwykle niewielki wpływ na modelowanie, to dla materiałów złożonych sprawa już nie jest tak oczywista. W pracy przeanalizowano płaskie płyniecie ma-

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teriału kompozytowego składającego się z trzech warstw plastycznych w matrycy klinowej. Pokazano, że granica podziału pomiędzy materiałami a nawet samo istnienie rozwiązania może istotnie zależeć od wyboru kryterium uplastycznienia. Zbadane zostało kryterium w najbardziej ogólnej postaci (wspomniane klasyczne kryteria uplastycznienia są przypadkami szczegółowymi). Wynik analizy mogą być stosowane do oceny dokładności modelowania inżynierskiego procesów wyciskania czy ciągnienia materiałów kompozytowych opierających się na poszczególnych kryteriach uplastycznienia stosowanych składników kompozytu.

1. Introduction

The position of bi-material interface in composite material essentially depends on its properties and conditions of plastic deformation are very important from the point of view of engineering practice.

Extrusion of bi-metallic longitudinally oriented composite is a more complicated process in comparison with mono-material flow in the same process. As it has been shown in [13] the position of the bi-material interface in the process depends on many factors. Moreover, even in the case of the simplest modeling of axisymmetric extrusion (radial flow field) the following additional problems should be considered: 1) the shape of the dead zone (in the sleeve mainly) and 2) the position and shape of the bi-material interface whose form depends on process parameters (Fig. 1). An additional problem in the modeling is connected with the fact that the friction regime at the bimaterial interface is unknown a priory. Both the regimes (sticking or sliding) may occur depending on process parameters (see Fig. 2 from [13]).

In the modelling of the aforementioned processes, the assumption of the yield criterion (e.g. Tresca or Huber- Mises) is very important because of the possible variation of interface position [2, 3]. Up to now there are no adequate rules to choose the criterion for different materials and under various conditions of plastic flow. For real process and its modelling it is very important to know the consequence of the choice of material description and yield criterion which has a great effect on the prediction of the position of the material interface.

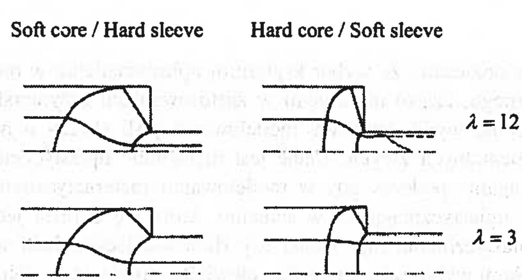

Fig. 1. Changing in the shape and position of bimaterial interface in plastic zone of Al/Pb composite (after extrusion of the bullet)

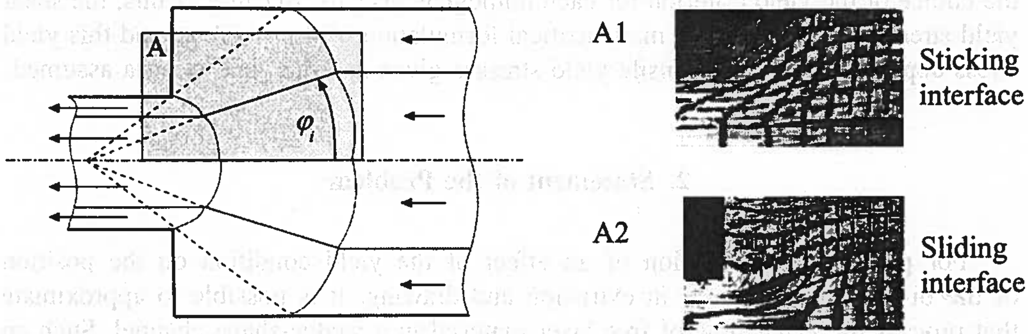

Fig. 2. Extrusion of bimetallic composite of sleeve-core system

The simplest analytical modeling has been done in papers [7, 8] under the simplified assumptions: both the materials are perfectly plastic, the flow is radial within the each material.

Flow in converging channels is a classical problem of solid and fluid mechanics. A solution for planar flow of rigid perfectly plastic material through a converging wedge- shaped channel has been given in Hill [8]. Assuming different yield criteria, the problem of axisymmetric flow of rigid/perfectly plastic material through a conical channel has been studied by Sokolovskii [14], Shield [12] and Alexandrov and Barlat [1]. Browman [4] has included rotation in the solution obtained by Sokolovskii. Planar and axisymmetric flows of rigid plastic, linear hardening materials have been studied by Durban and Budianski [6] and Durban [5], respectively. Malinin [10] has considered planar and axisymmetric flows of viscous materials. All of the aforementioned solutions have dealt with homogeneous materials. However, piece- wise homogeneous materials are widely used in modern industry. A typical process for forming such materials is extrusion (for example, Sliwa, [13]). Solutions for flow through converging dies can serve as a basis for approximate analysis of the extrusion process. Several aspects of the plane-strain theory of plasticity for piece-wise homogeneous materials have been studied by Rychlewski [11]. Solutions for planar and axisymmetric flows of multi-layer materials through wedge-shaped and conical channel have been obtained in Durban [7], Alexandrov *et.al.*. [3] and Alexandrov and Mishuris [2]. In these solutions, it has been assumed that the materials obey the same yield criterion such that the only difference in the constitutive behaviour is the magnitude of the yield stress in tension. In the present paper, an effect of the choice of the yield criterion on solution behaviour for planar flow of three-layer material through a wedge-shaped channel is investigated. The analysis is based on the solution given in Alexandrov *et.al.*. [3]. In the case of plane-strain problems for the rigid/perfectly plastic homogeneous materials, the solution in dimensionless form is independent of the yield criterion. However, when piece-wise homogeneous rigid plastic material deforms and yielding of each homogeneous part is determined by the corresponding tensile yield stress, the solution behaviour depends on the choice of the yield criterion for each homogeneous part. Because of this, the shear yield stress is involved in the mathematical formulation of the problem, and this yield stress depends on both the tensile yield stresses given and the yield criteria assumed.

2. Statement of the Problem

For preliminary evaluation of an effect of the yield condition on the position of the bi- material interface in extrusion and drawing, it is possible to approximate that process by planar flow of free layer material in a wedge-shape channel. Such an approach permits for a quick design of the process.

Consider a converging wedge-shaped channel (total angle 2α) through which a three-layer strip is being forced (Fig. 3). The layers adjacent to the walls are made of the same material. Thus the problem is symmetric with respect to the horizontal axis passing through the apex of the channel. Introduce a polar coordinate system, $r\theta$, with its origin coinciding with the apex. Because of symmetry, only the space $\theta \geqslant 0$ needs to be considered. Each material is rigid perfectly plastic. Sliding occurs at the bi- material interface, $\theta = \theta_i$. The value of θ_i is unknown and should be found from the solution. Friction at the wall and the bi-material interface is described by the constant friction law with the friction factors m_0 and m_i, respectively.

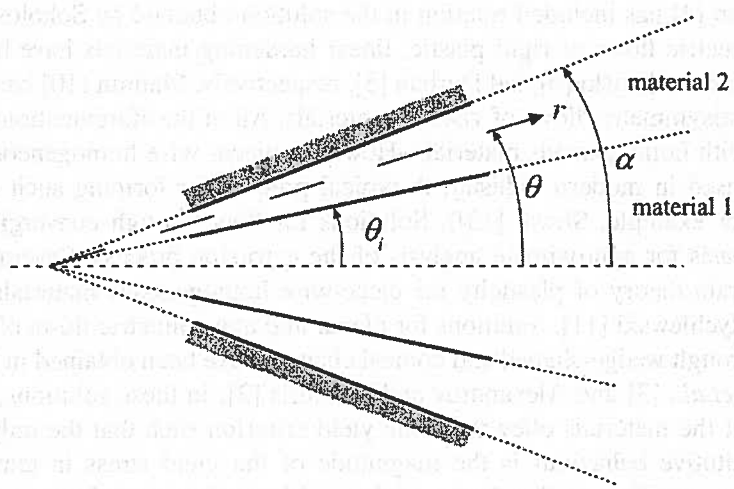

Fig. 3. Notation for three-layer flow through converging wedge-shaped channel

The tensile yield stresses, $\sigma_Y^{(1)}$ and $\sigma_Y^{(2)}$, are supposed to be known from experiment. Here and in what follows the superscripts (1) and (2) denote the material 1 and 2, respectively, as shown in Fig. 3. The yield criterion for each material should be chosen by specifying n in the generalized isotropic yield criterion proposed by Hosford (1972).

In terms of the principal stresses, σ_1, σ_2 and σ_3, the latter has the form

$$\left[\frac{(\sigma_1-\sigma_2)^{n^{(i)}}+(\sigma_2-\sigma_3)^{n^{(i)}}+(\sigma_1-\sigma_3)^{n^{(i)}}}{2}\right]^{1/n^{(i)}}=\sigma_Y^{(i)},\quad i=1,2, \tag{1}$$

with some real value $n \geqslant 1$. In general, $n^{(1)} \neq n^{(2)}$. The shear yield stress is expressed in the form

$$\tau_Y=\sigma_Y\left(2^{n-1}+1\right)^{-1/n}. \tag{2}$$

Using (2) the ratio of the shear yield stresses of the two materials involved in the solution given in [3] can be written as

$$\tau=\frac{\tau_Y^{(1)}}{\tau_Y^{(2)}}=\tau_e\tau_a, \tag{3}$$

where

$$\tau_e=\frac{\sigma_Y^{(1)}}{\sigma_Y^{(2)}},\quad \tau_a=\frac{t^{(2)}}{t^{(1)}}\quad \text{and}\quad t^{(i)}=(2^{n^{(i)}-1}+1)^{1/n^{(i)}},\quad \mathrm{i}=1,2. \tag{4}$$

The value of τ_e is supposed to be known and fixed whereas the value of τ_a is uncertain since in many cases the experimental output is limited to the tensile yield stress. The main objective of this paper is to investigate an effect of the value of τ_a on solution behaviour. The value of n involved in (1) can vary in the interval

$$1 \leqslant n < \infty. \tag{5}$$

The Tresca yield criterion is obtained at $n=1$ and $n=\infty$, and the Huber-Mises yield criterion at $n=2$ and 4. In the interval (5), t considered as the function of n attains its maximum at $n=1$ where $t=2$, and minimum at $n \approx 2.767$ where $t \approx 1.71$. Therefore τ_a may vary in the interval

$$\frac{1.71}{2} \leqslant \tau_a \leqslant \frac{2}{1.71}. \tag{6}$$

3. Solution

The solution for arbitrary τ is described in detail in [3]. Its structure depends on τ, m_0 and m_i. Also it depends on the direction of sliding at the bi-material interface. Here, with the use of (3), the final equations to be solved are summarized for most important cases.

3.1. The velocity of material 1 is higher than the velocity of material 2 at the bi-material interface and $\tau_a\tau_e > 1$

Using (3) the system of equations for c, θ_0 and θ_i given in [3] takes the following form

$$\theta_i = -\psi_i^{(1)} + \frac{c}{\sqrt{c^2-1}} \arctan\left[\sqrt{\frac{c+1}{c-1}} \tan\psi_i^{(1)}\right],$$
$$\theta_i = -\psi_i^{(2)} + \frac{\tau_a\tau_e c}{\sqrt{\tau_a^2\tau_e^2c^2-1}} \arctan\left[\sqrt{\frac{\tau_a\tau_e c+1}{\tau_a\tau_e c-1}} \tan\psi_i^{(2)}\right] + \theta_0, \tag{7}$$
$$\alpha = -\psi_\alpha^{(2)} + \frac{\tau_a\tau_e c}{\sqrt{\tau_a^2\tau_e^2c^2-1}} \arctan\left[\sqrt{\frac{\tau_a\tau_e c+1}{\tau_a\tau_e c-1}} \tan\psi_\alpha^{(2)}\right] + \theta_0,$$

where $\psi_i^{(1)}$, $\psi_i^{(2)}$ and $\psi_\alpha^{(2)}$ are determined from

$$\tau_a\tau_e \sin 2\psi_i^{(1)} = \sin 2\psi_i^{(2)} = m_i \qquad \text{and} \qquad \sin 2\psi_\alpha^{(2)} = m_0. \tag{8}$$

Of three unknowns involved in (7), c and θ_0 are auxiliary parameters and θ_i is a quantity of practical interest, the position of the bi-material interface (Fig. 3). Therefore, in the next section the latter quantity is used to illustrate the main features of this and following solutions.

3.2. The velocity of material 1 is higher than the velocity of material 2 at the bi-material interface and $\tau_a\tau_e < 1$

3.2.1. The case $m_i\tau_a\tau_e < m_0$

The structure of the solution in this case depends on the value of $\tau_a\tau_e c$. If $\tau_a\tau_e c > 1$, the solution reduces to equations (7). If $\tau_a\tau_e c = 1$, c, θ_0 and θ_i should be found from

$$\theta_i = -\psi_i^{(1)} + \frac{c}{\sqrt{c^2-1}} \arctan\left[\sqrt{\frac{c+1}{c-1}} \tan\psi_i^{(1)}\right],$$
$$\theta_i = -\psi_i^{(2)} - \frac{1}{2}\cot\psi_i^{(2)} + \theta_0, \tag{9}$$
$$\alpha = -\psi_\alpha^{(2)} - \frac{1}{2}\cot\psi_\alpha^{(2)} + \theta_0$$

Finally, if $\tau_a \tau_e c < 1$, the solution reduces to the system of equation in the following form

$$\theta_i = -\psi_i^{(1)} + \frac{\cos 2\varphi}{\sqrt{\cos^2 2\varphi - \tau_a^2 \tau_e^2}} \arctan\left[\sqrt{\frac{\cos 2\varphi + \tau_a \tau_e}{\cos 2\varphi - \tau_a \tau_e}} \tan \psi_i^{(1)}\right],$$

$$\theta_i = -\psi_i^{(2)} + \frac{\cos 2\varphi}{2} \ln\left[\frac{\sin\left(\psi_i^{(2)} - \varphi\right)}{\sin\left(\psi_i^{(2)} + \varphi\right)}\right] + \theta_0, \tag{10}$$

$$\alpha = -\psi_\alpha^{(2)} + \frac{\cos 2\varphi}{2} \ln\left[\frac{\sin\left(\psi_\alpha^{(2)} - \varphi\right)}{\sin\left(\psi_\alpha^{(2)} + \varphi\right)}\right] + \theta_0,$$

where φ is defined by $\cos 2\varphi = c\tau_a \tau_e$. It is possible to show that at given values of α, m_0, m_i, τ_a, and τ_e one and only one of the systems of equations (7), or (9) or (10) has a solution.

3.2.2. The case $m_i \tau_a \tau_e > m_0$

In this case, the solution, if it exists, reduces to the system of equations (10).

3.3. The velocity of material 2 is higher than the velocity of material 1 at the bi-material interface

In this case the solution may exist if and only if $c\tau_a \tau_e > 1$. The system of equations for ϕ, θ_0 and θ_i reduces to

$$\theta_i = -\psi_i^{(1)} + \frac{\cos 2\phi}{2} \ln\left[\frac{\sin(\phi - \psi_i^{(1)})}{\sin(\phi + \psi_i^{(1)})}\right],$$

$$\theta_i = -\psi_i^{(2)} + \frac{\tau_a \tau_e \cos 2\phi}{\sqrt{\tau_a^2 \tau_e^2 \cos^2 \phi - 1}} \arctan\left[\sqrt{\frac{\tau_a \tau_e \cos 2\phi + 1}{\tau_a \tau_e \cos 2\phi - 1}} \tan \psi_i^{(2)}\right] + \theta_0, \tag{11}$$

$$\alpha = -\psi_\alpha^{(2)} + \frac{\tau_a \tau_e \cos 2\phi}{\sqrt{\tau_a^2 \tau_e^2 \cos^2 \phi - 1}} \arctan\left[\sqrt{\frac{\tau_a \tau_e \cos 2\phi + 1}{\tau_a \tau_e \cos 2\phi - 1}} \tan \psi_\alpha^{(2)}\right] + \theta_0,$$

where ϕ is defined by $\cos 2\phi = c$.

3.4. Qualitative and quantitative effects of the yield condition on the solution

3.4.1. The velocity of material 1 is higher than the velocity of material 2 at the bi-material interface (Sections 3.1 and 3.2)

The case $\tau_e = 1$. In this case the experimental tensile yield stress is the same for both materials. Therefore, if the same yield condition is assumed for the materials, the unique solution corresponding to the flow of homogeneous material is obtained (except that there may be a velocity jump at the bi-material interface). However, materials may obey different yield criteria even though the tensile yield stress is the same. The yield condition assumed has a great effect on the solution. The solution to equations (7), (9) and (10) has been found numerically in the interval (6). It is illustrated in Fig. 4. The friction factors are $m_1 = 0.5$ and $m_0 = 1.0$.

The case $\tau_e = 0.1912$. The ratio of the tensile yield stresses corresponding to a composition "Aluminum – Lead" is 0.1912 (Sliwa, [13]). In the case under consideration the layer made of lead is between two layers made of aluminum. The solution to equations (7), (9) and (10) has been found numerically in the interval (6). It is illustrated in the same Fig. 4 for the same friction parameters.

The case $\tau_e = 5.23$. In the this case, the same materials are considered but the layer made of aluminum is between two layers made of lead. The solution to equations (7), (9) and (10) has been found numerically in the interval (6). It is illustrated in Fig 4 for $m_i = 0.5$ and $m_0 = 1.0$.

It is easy to observe (Fig. 4) that the influence of the yield criterion assumed on the value of θ_i is most significant for $\tau_e = 1$. In Fig. 5, the same results are presented for θ_i normalized by its value obtained from the solution in which the same yield criterion is assumed for both materials.

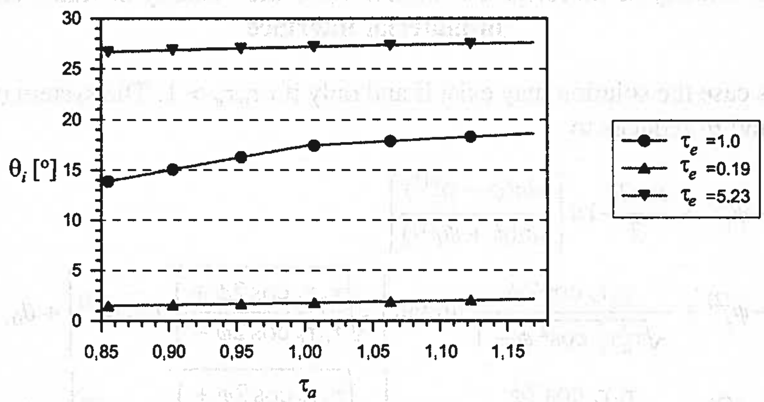

Fig. 4. Variation of angle θ_i determining the position of the bi-material interface with τ_a (logarithmic scale) at different values of τ_e

As one can conclude from these results, smaller influence appears when the external material is stiffer then the inner one. It is important to find the range of parameter τ_e where the highest sensitivity of the value of normalized θ_i to the criterion chosen occurs. To this end it is necessary to investigate the ratio $\theta_i(\tau_e\tau_a)/\theta_i(\tau_e)$ where the value $\theta_i(\tau_e) = \theta_i^*$ corresponds to $\tau_a = 1$. Numerical results are illustrated in Fig. 6 where τ_e varies but τ_a takes the end points of the interval (6).

The friction factors are $m_i = 0.5$ and $m_0 = 1.0$. Most significant deviations from the case $\tau_a = 1$ occur in the interval $\tau_e \in (0.1, 1.2)$, and for $\tau_e \approx 0.7$ the maximum

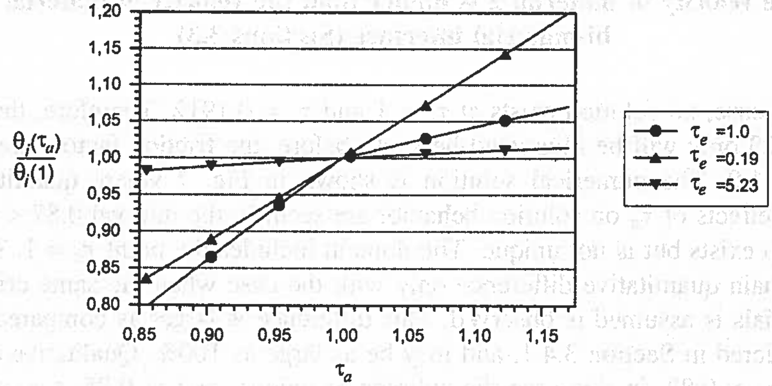

Fig. 5. Variation of normalized angle θ_i with τ_a(logarithmic scale) at different values of τ_e

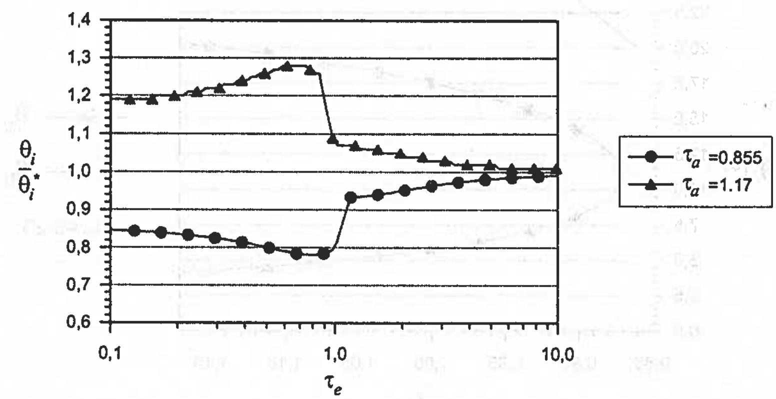

Fig. 6. Influence of τ_e (logarithmic scale) on deviation of θ_i from its value at $\tau_a = 1$ for two extreme cases

deviation due to the choice of the yield criterion is in the range — 22% to 28%. This fact may have a significant effect on the interpretation of the solution for practical applications. It is important to mention that typical experimental values of τ_e belong to the interval (0.1,1.2).

The only case where the unique solution exists has been investigated here and, therefore, the effect of the yield criterion on solution behavior is solely quantitative. If the domain of problem parameters where two solutions or no solution exists is included into consideration, qualitative differences in solution behavior occur. This feature of solution behavior will be illustrated for the case described in Section 3.3.

3.4.2. The velocity of material 2 is higher than the velocity of material 1 at the bi-material interface (Sections 3.3)

In this case, no solution exists at $\tau_e = 1$ and $\tau_e = 0.1912$. Therefore, the solution for $\tau_e = 5.23$ only will be illustrated here. As before, the friction factors are $m_i = 0.5$ and $m_0 = 1.0$. The numerical solution is shown in Fig. 7 where quantitative and qualitative effects of τ_a on solution behavior are seen. In the interval $0.87 < \tau_a < 1.17$ the solution exists but is not unique. The domain includes the point $\tau_a = 1$. Therefore, in this domain quantitative difference only with the case when the same criterion for both materials is assumed is observed. This difference is larger as compared with the case considered in Section 3.4.1. and may be as large as 100%. Qualitative difference occurs at $\tau_a \approx 0.87$, in this case the solution is unique, and at $0.85 < \tau_a < 0.87$ no solution exists.

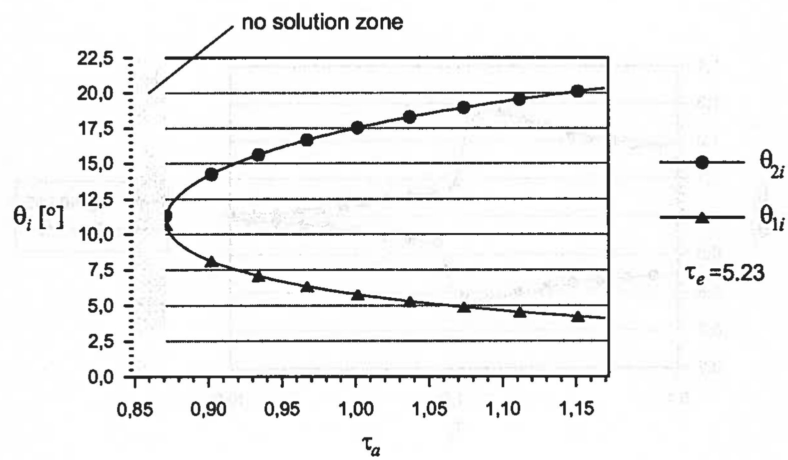

Fig. 7. Influence of τ_a (logarithmic scale) on solution behavior

3.4.3. Frictionless interface

As in the case considered in Section 3.4.2., the solution with no friction at the bi-material interface exists only if $\tau_e = 5.23$ (of the values of τ_e considered in this paper). However, in contrast to the case of Section 3.4.2., the solution is unique such that quantitative difference occurs only. In particular, the value of θ_i determining the position of the bi-material interface is presented in Fig 8 for different values of the friction factor, namely $m_0 = 0.1; 0.5; 1.0$. The most significant influence appears for the highest value of the friction factor.

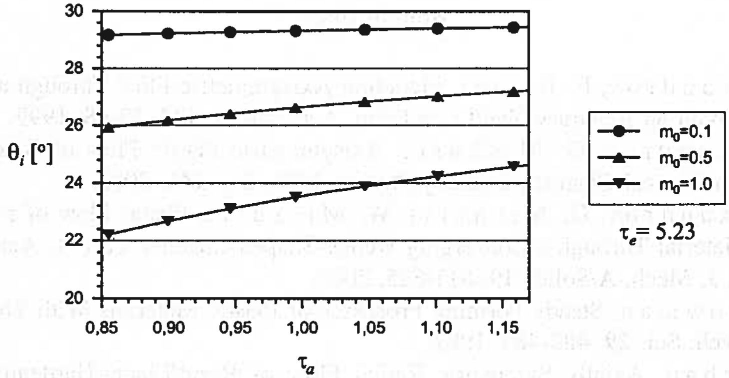

Fig. 8. Influence of τ_a (logarithmic scale) on solution behavior in the case of frictionless bi-material interface

4. Conclusions

The system of equations of the plane-strain rigid/perfectly plastic model for homogeneous materials is independent of the yield criterion and, in dimensionless form, of the magnitude of the yield stress. When piece-wise homogeneous materials are considered, the ratios of the shear yield stresses influences solutions whereas the yield criterion has no effect. However, in practice, the experimental shear yield stress is usually unknown but the tensile yield stress. Therefore, a yield criterion should be chosen to transform the tensile yield stress into the shear yield stress involved in the mathematical formulation of problems. In the case of homogeneous materials, the only effect of it is the magnitude of stress and load. However, for piece-wise homogeneous materials the yield criteria may be different for different homogeneous parts of the piece-wise homogeneous material. In this case, the choice of each yield criterion may have a significant quantitative and qualitative effect on the solution and, as a result, on its interpretation for engineering applications. It has been demonstrated for the problem of planar flow of three-layer material through a convergent wedge-shape channel. In particular, an effect of the choice of yield criterion on the position of the bi-material interface has been studied for several combinations of materials and frictional boundary conditions of practical interest. In some cases, a significant quantitative difference (up to 100%), as compared with the situation where the same yield criterion is chosen for all material, has been found. In other cases, the difference is even qualitative (the solution is unique or multiple for one class of yield criteria and does not exist for another class).

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