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ALGORITHMS OF FURNACE CHARGE BURDEN OPTIMISATION IN FOUNDRIES

ALGORYTMY OPTYMALIZACJI NAMIAROWANIA W SYSTEMACH ZAŁADUNKU PIECÓW ODLEWNICZYCH

This article describes the methods helpful in choosing an optimum charge burden for foundry furnaces with classification of those techniques which can be useful in respective calculations. A basic part of all methods are mathematical models describing various situations which can happen in the technological process when charge burden is calculated. The primary parameter in all models is the chemical composition of charge constituents, which can be defined in either deterministic or fuzzy form. The secondary parameter adopted in calculations are the currently used unit prices of charge materials. The analysis and synthesis of the methods of charge calculation is completed with an example where the optimisation task of charge burden calculation has been solved using as a tool the author's own computer program.

Keywords: burden calculations, fuzzy optimisation, foundry furnaces

W artykule przedstawiono metodykę doboru optymalnego namiaru wsadu dla pieców odlewniczych wraz z klasyfikacją metod mogących mieć zastosowanie w obliczeniach. Integralną częścią metod są modele matematyczne ujmujące możliwe sytuacje w procesie technologicznym namiarowania wsadu. Parametrem podstawowym w poszczególnych modelach jest skład chemiczny składników, który może być zdefiniowany w postaci deterministycznej lub rozmytej. Parametrami drugorzędnymi, jakie przyjęto do obliczeń, są ceny jednostkowe zakupu materiałów wsadowych. Dopełnieniem analizy i syntezy metodyki wyznaczania namiaru jest przykład obliczeniowy, w którym zadanie optymalizacji rozwiązano, stosując autorski program komputerowy.

1. Introduction

In an attempt of optimising the casting production process it is necessary to use the most modern methods of planning and control at nearly all stages of the technological process. One of the stages being most important in the manufacture of castings is fabrication of liquid metal characterised by strictly determined parameters. Most frequently these parameters are dictated by the type of the applied technology of manufacture, including the quantity (weight) of charge, chemical composition as well as, and tapping and pouring temperature. A very important factor in optimisation of the economic production of castings is also the cost of fabrication of a mass volume of molten metal which, under given conditions, should be as low as possible. To satisfy the above requirements, various attempts have been made to determine optimum charge burden, adjusted to the type of foundry furnace currently used and to the adopted policy of management of the charge materials stored in foundry.

The article describes the classification of the methods of charge burden calculation, with selected mathematical models of tasks for optimum charge burden computation. The principles of modelling the uncertain (fuzzy) values, which are used in the description of inaccurate chemical composition and/or unit price of individual charge materials have been described.

2. Classification of methods used to calculate the charge burden

It follows from various sources of information known to the author that the reference literature (Polish and international) lacks a study which would discuss in a comprehensive way the theory of charge burden calculation. Some fundamentals are only available, and they can serve as a background against which a theory like

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the one described in this article can be constructed. The main element in this theory will be the method of charge burden calculation. The author proposes a classification of the methods used so far for charge burden calculation according to the following criteria:

I. Melting process
1. Calculation of charge burden to fabricate molten metal:
   a) Calculation of charge burden for an empty furnace
   b) Calculation of charge burden for a furnace partially filled with:
      – solid charge
      – melted charge.
2. Calculation of charge burden to correct the chemical composition of molten metal in furnace (or in foundry ladle):
   – simplified methods,
   – optimising methods.

II. Type of method used in charge burden calculation
1. Diagrammatic methods:
   – graphical methods,
   – geometric methods.
2. Analytical methods
3. Numerical methods (computer-aided methods):
   – linear algebry,
   – optimisation
      • methods of linear programming
      • methods of square programming

III. Type of charge and values of cast alloy parameters (chemical composition, unit price):
1. Deterministic description
2. Fuzzy description:
   – two-level,
   – multi-level with finite number of levels,
   – continuous.

IV. Type of optimisation:
1. Monocriterial tasks of charge burden calculation
2. Multicriterial tasks of charge burden calculation

The presented classification allows for the conventional methods of charge burden calculation, taking also into consideration the most modern techniques of fuzzy optimisation developed by the author [1-4].

3. Stages of charge burden calculation

Calculation of charge burden to each type of the foundry furnace consists of the three basic stages:

Stage I – Determining, from technological guidelines or by calculations, the weight of the charge unit. In most cases, the weight of the charge depends on foundry's production demand and is restricted by the rated capacity of melting installations. Only in the case of cupolas, the empirical formulae are used, and they enable the weight of a single charge burden to be effectively determined [1, 2].

Stage II – Determining, by assumption or calculation, the chemical composition of charge which, for a given proces of melting, will ensure the correct chemical composition of molten metal.

Stage III – Determining, by the selected method, the charge burden. This operation includes calculation of percent or mass fraction of charge components included in the total calculated charge.

The decisions of technological nature taken at Stage II have an important effect on final output of the melting process. This mainly refers to the chemical composition of molten metal which may also depend on factors other than the sole chemical composition. The factors that are responsible for the chemical composition of molten metal include the type of charge melting process and the type of charge melting installation. It is generally assumed that in electric furnaces (induction, arc and resistance), because of melting losses, the content of chemical elements in molten metal is lower than it is in the charge. In cupolas this situation is much more complicated, mainly because the content of carbon and sulphur tends to increase, since the metallic charge is melted in the presence of a solid fuel (coke), containing these both elements.

The choice of a method to carry out Stage III depends on how complex the task of charge burden calculation is, i.e. on the number of chemical elements and charge components included in calculations. The simplest tasks including one, two or three chemical elements can be solved by means of a selected diagrammatic method (graphical or geometric) or, if possible, can be reduced to a system of linear equations and solved by one of the analytical or numerical methods of the linear algebra (for example, Gauss or Gauss-Jordan elimination method).

Further part of this publication describes in a more comprehensive way the mathematical models used in determination of optimum charge burden.

4. A generalised deterministic model of charge burden calculation

One of the first mathematical models of the task of charge burden calculation was described in [1]. The study describes the task of minimising, where the objective function is assuming the following form:

$$f(\mathbf{x}) = \sum_{j=1}^{N} c_j x_j \tag{1}$$

where:

x – vector of the searched decision variables; in this case it is the content of each charge component in % (if $m_w = 100\%$) or in kg (if m_w is expressed in kg),

N – number of charge materials considered in calculations,

c_j – unit price of j-th charge material in price unit per mass unit.

The objective function (1) searches for the charge burden of the lowest total cost. The technological guidelines for this model were defined as a system of constraints:

$$\begin{cases} \sum_{j=1}^{N} a_{ij} x_j = b_i \cdot m_w \\ \sum_{j=1}^{N} x_j = m_w \\ 0 \leqslant x_j \leqslant \bar{x}_j \\ i = 1, 2, ..., M; j = 1, 2, ..., N \end{cases} \tag{2}$$

where:

a_{ij} – content of i-th chemical element in j-th charge component, %,

b_i – content of i-th chemical element in charge, %,

M – number of chemical elements considered in calculations,

m_w – charge weight in kg or equal to 100%,

$\bar{x}_j$ – upper limit of the content of j-th component in charge in kg, or in %.

In the quoted reference literature [1], five basic chemical elements were taken into consideration, i.e. carbon, silicon, manganese, phosphorus and sulphur (C, Si, Mn, P and S). Additionally, the effect of constraints on the content of individual materials in the calculated charge has been assumed. The task of optimisation of the objective function (1) with contraints (2) is the task of linear optimisation, since both function (1) as well as the system of constraints (2) have the form of linear functions. In solving thus formulated task one can use, e.g., the method of simplexes [8, 9].

The optimisation task (1÷2) may create some problems of strictly numerical nature. The system of constraints (2) does not allow for the lower limits in content of each of the considered charge materials, so willingly used by process engineers preparing the metal melting process. If strictly determined chemical composition of charge represented by a value b_i is used, it may cause errors when the values calculated with very high accuracy by computer are rounded, making determination of the value of vector **x** impossible, which the program recognises as a lack of solution for the optimisation task under consideration. To eliminate the above mentioned inconveniences, it would be much better and easier to adopt a system of constraints in the form [6]:

$$\begin{cases} \underline{b}_i m_w \leqslant \sum_{j=1}^{N} a_{ij} x_j \leqslant \bar{b}_i m_w i = 1, 2, ..., M \\ 0 \leqslant \underline{x}_j \leqslant x_j \leqslant \bar{x}_j \leqslant m_w, j = 1, 2, ..., N \\ \sum_{j=1}^{N} x_j = m_w \end{cases} \tag{3}$$

where:

$\underline{b}_i$, $\bar{b}_i$ – the lower and upper limit, respectively, in the content of i-th chemical element in charge, %,

$\underline{x}_j$, $\bar{x}_j$ – the lower and upper limit, respectively, in the content of j-th charge material, expressed in the same units as x_j.

Then, the task of computation of the charge burden consists in searching for a set of values of vector x such that will enable minimising the value of function (1) under constraints (3). Yet, this task continues being the task of linear optimisation, which means that it is possible to use one of the methods of linear programming.

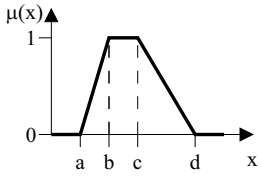
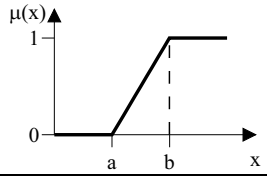
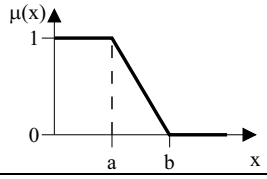
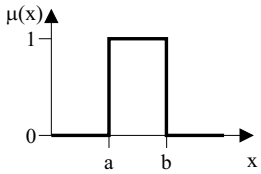
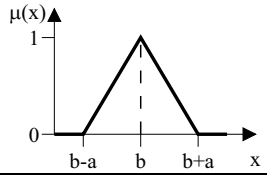
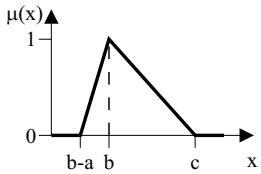
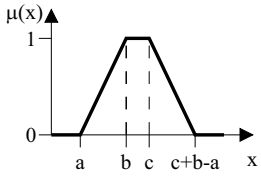
5. A generalised fuzzy model of charge burden calculation

Applying the deterministic (strictly determined) values of parameters, such as the chemical composition or unit price of charge materials is usually due to:

- averaging the values of the considered parameters, due to fuzzy laboratory measurements of the chemical composition done on a large volume of charge materials and simplified data comprised in quality certificates,
- disregarding some features (chemical composition, unit price) common to different batches of the same component, collected in the same bin,
- constraints imposed by mathematical models used so far to describe the optimisation tasks, and difficult to access computer programs used for solving of these tasks.

TABLE 1

Compilation of basic polygonal membership functions composed of rectilinear segments

Name of polygonal membership function	Plotted graph	Definition of membership function
Asymmetric trapezoid		$\mu(x)=\begin{cases}0 & for\ x<a\ or\ x\geq d\\ \frac{x-a}{b-a} & for\ a\leq x<b\\ 1 & for\ b\leq x<c\\ \frac{d-x}{d-c} & for\ c\leq x<d\end{cases}$
Left outside		$\mu(x)=\begin{cases}0 & for\ x<a\\ \frac{x-a}{b-a} & for\ a\leq x\leq b\\ 1 & for\ x>b\end{cases}$
Right outside		$\mu(x)=\begin{cases}1 & for\ x<a\\ \frac{b-x}{b-a} & for\ a\leq x\leq b\\ 0 & for\ x>b\end{cases}$
Rectangular		$\mu(x)=\begin{cases}0 & for\ x<a\ or\ x>b\\ 1 & for\ a\leq x\leq b\end{cases}$
Symmetric triangle		$\mu(x)=\begin{cases}0 & for\ x<(b-a)\ or\ x>(b+a)\\ \frac{a-\lvert x-b\rvert}{a} & for\ (b-a)\leq x\leq(b+a)\end{cases}$
Asymmetric triangle		$\mu(x)=\begin{cases}0 & for\ x<(b-a)\ or\ x>(b+a)\\ \frac{a+x-b}{a} & for\ (b-a)\leq x<b\\ \frac{c-x+b}{a} & for\ b\leq x\leq(b+a)\end{cases}$
Symmetric trapezoid		$\mu(x)=\begin{cases}0 & for\ x<a\ or\ x>(c+b-a)\\ \frac{x-a}{b-a} & for\ a\leq x<b\\ 1 & for\ b\leq x<c\\ \frac{d-x}{d-c}=\frac{c+b-a-x}{b-a} & for\ c\leq x\leq(c+b-a)\end{cases}$

5.1. Mathematical description of uncertain (fuzzy) values

In situation when the precisely determined values of parameters cause oversimplifications considerably restricting their practical use, one can recur to a description of the investigated domain of reality using the theory of fuzziness. The uncertainty of the values of the numerical variables can be described by means of fuzzy sets [10]. According to [10], by the name of fuzzy set A, in a numerical space of considerations X, we denote a set of pairs:

$$A=\{(\mu_A^*(x),x)\},\forall x\in X \tag{4}$$

where:

μ_A – membership function of fuzzy set A, which to every element $x \in X$ ascribes the degree of its membership μ_A^* in fuzzy set A, where $\mu_A(x) \in [0; 1]$.

The membership function ascribes to each element x of a given variable a value from the domain [0;1]:

$$\mu_A(x) : X \rightarrow [0; 1], \forall x \in X \tag{5}$$

The value, called the degree of membership, informs us to what degree element x belongs to a fuzzy set A. In [10] a comprehensive review of fuzzy sets and the related knowledge have been presented.

For the description of uncertain (inaccurate), i.e. fuzzy, values of parameters characteristic of the chemical composition and possibly also of the unit price of charge materials, one can use the membership functions of different classes. Table 1 below gives compilation of the polygonal membership functions built from rectilinear segments.

It has been decided to describe the fuzziness of the chemical composition of charge materials by a polygonal function in the form of uneven-armed trapezoid (Fig. 1). The choice has been justified in the following way:

a) the membership function in the form of asymmetric trapezoid operates at a level "0", while the remaining two functions operate at a level "1". In foundry practice, describing the fuzziness of chemical composition at more than two levels of a membership function significantly increases the dimension of the optimisation task, which finds no justification in practical application. It can be assumed that level "0" is a "pessimistic" level, i.e. such for which the range of the content of chemical elements will be the least favourable. On the other hand, level "1" can be called "optimistic", i.e. the one for which the range of the content of chemical elements will be the narrowest.

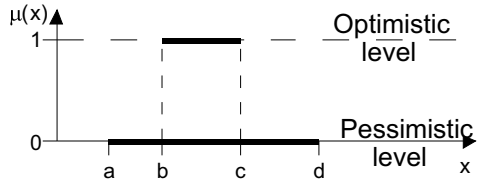


Fig. 1. Graphic interpretation of the content ranges of chemical elements at the levels: μ(x)= 0 ("pessimistic") and μ(x)= 1 ("optimistic") for a membership function in the form of asymmetric trapezoid

b) in terms of a fuzzy description of the chemical composition of charge components, the function of uneven-armed trapezoid replaces all other polygonal functions of membership,

c) the function is characterised by four numbers, which make the technological identification very easy, and is very handy in collecting and processing of information in the database of charge materials. The first number denotes the lower content of chemical element at a "pessimistic" level. The second number and the third number denote, respectively, the lower and upper contents of this element at an "optimistic" level, while the last, fourth number denotes the upper content of chemical element at a "pessimistic" level.

The proposed model of description of the fuzziness is also applicable when describing the uncertain unit prices of charge materials allowed for the calculations.

5.2. Model of charge burden calculation for fuzzy chemical composition of charge materials and determined unit prices

The task of foundry furnace charge burden calculation allowing for a fuzzy chemical composition of charge materials will consist in determination of such fractions of x_j, for which the value of the objective function will be minimised to the form:

$$\min_{\mathbf{x}} \sum_{j=1}^{N} c_j x_j \tag{6}$$

under the following constraints:

$$\begin{cases} \sum_{j=1}^{N} \underline{a}_{ijk} x_j \geqslant \underline{b}_{ik} m_w \\ \sum_{j=1}^{N} \bar{a}_{ijk} x_j \leqslant \bar{b}_{ik} m_w \\ 0 \leqslant \underline{x}_j \leqslant x_j \leqslant \bar{x}_j \leqslant m_w \\ \sum_{j=1}^{N} x_j = m_w \\ i = 1, 2, ..., M; j = 1, 2, ..., N \\ k = 1, 2, ..., q \end{cases} \tag{7}$$

where:

q – number of the membership function levels taken into consideration.

For a rectangular model of the description of fuzziness ($q = 2$), the system of constraints will assume the form of:

$$\begin{cases} \sum_{j=1}^{N} \underline{a}_{ij} x_j \geqslant \underline{b}_i \cdot m_w \\ \sum_{j=1}^{N} \bar{a}_{ij} x_j \leqslant \overline{b}_i \cdot m_w \\ 0 \leqslant \underline{x}_j \leqslant x_j \leqslant \bar{x}_j \leqslant m_w \\ \sum_{j=1}^{N} x_j = m_w \end{cases} \tag{8}$$

while for a trapezoidal model, the constraints in optimisation task can be defined in the following way:

$$\begin{cases} \sum_{j=1}^{N} \underline{a}_{ij0} x_j \geqslant \underline{b}_{i0} \cdot m_w \\ \sum_{j=1}^{N} a_{ij1} x_j \geqslant \underline{b}_{i1} \cdot m_w \\ \sum_{j=1}^{N} \bar{a}_{ij0} x_j \leqslant \overline{b}_{i0} \cdot m_w \\ \sum_{j=1}^{N} \bar{a}_{ij1} x_j \leqslant \bar{b}_{i1} \cdot m_w \\ 0 \leqslant \underline{x}_j \leqslant x_j \leqslant \bar{x}_j \leqslant m_w \\ \sum_{j=1}^{N} x_j = m_w \end{cases} \tag{9}$$

5.3. Model of charge burden calculation for fuzzy chemical composition of charge materials and fuzzy unit prices

The charge burden with strictly determined unit price of charge materials can be calculated when these materials have already been purchased by the foundry, or when their purchase is planned and it is known for sure that both the terms of purchase as well as the parameters shall not change [5]. In the situation of long-term planning of charge materials purchase, or when sudden changes of prices over short periods of time threaten, a very convenient option may be the possibility of determining a charge burden for uncertain prices of the charge components.

The fuzziness of the price of charge materials can be simulated with the help of selected membership functions in a way similar as it happens in the case of fuzzy chemical composition.

If the price of a charge component is explicitly determined, then the objective function in optimisation task cannot be defined in accordance with relationship (1). In this case, for each level of the membership function describing the price fuzziness, a separate objective function should be applied. If so, then there are many objective functions which reduce the optimisation task to a multi-criterial linear programming under the assumption that the system of constraints is in form (2) or in one of its variants, e.g. (3) or (5).

One of the possibilities to determine the value of vector x, for which all the linear objective functions are to be minimised, is by defining the, so called, compromise objective function. In its general form, this objective function can be described as:

$$\min_{\mathbf{x}} \left[\sum_{k=1}^{q} \left[\left(\underline{f}_k - \sum_{j=1}^{N} \underline{c}_{jk} x_j \right)^2 + \left(\bar{f}_k - \sum_{j=1}^{N} \bar{c}_{jk} x_j \right)^2 \right] \right], \tag{10}$$

where:

q – maximum quantity of membership function degrees for each of the fuzzy numbers allowed for in the models of fuzziness,

$\underline{f}_k$ – the value of objective function in an optimisation task in which the objective function at k-th level has the form of $\min_{\mathbf{x}} \left[\underline{f}_k = \sum_{j=1}^{N} \underline{c}_{jk} x_j \right]$ under constraints (2)

$\overline{f}_k$ – the value of objective function in an optimisation task in which the objective function at k-th level has the form of $\min_{\mathbf{x}} \left[\bar{f}_k = \sum_{j=1}^{N} \bar{c}_{jk} x_j \right]$ under constraints (2)

Thus formulated objective function should be regarded as an effort to determine the value of vector x, i.e. a charge burden the total cost of which will approach both minimum and maximum values of unit prices of the individual charge materials.

To define the objective function (10), it is necessary to minimise, at each k-th level, the total cost of charge, assuming the minimum, first, and maximum, next, values of the cost of each charge component. This enables determination of the value of coefficients $\underline{f}_k$ and $\overline{f}_k$ and substituting them in relationship (10).

For a rectangular model of fuzziness, the compromise objective function shall assume the form of:

$$\min_{\mathbf{x}} \left[\left(f_0 - \sum_{j=1}^{N} \underline{c}_j x_j \right)^2 + \left(f_1 - \sum_{j=1}^{N} \bar{c}_j x_j \right)^2 \right], \tag{11}$$

where

$$f_0 = \min_{\mathbf{x}} \left[\sum_{j=1}^{N} \underline{c}_j x_j \right] \quad \text{and} \quad f_1 = \min_{\mathbf{x}} \left[\sum_{j=1}^{N} \bar{c}_j x_j \right], \tag{12}$$

while for the trapezoidal model of fuzziness it shall assume the form of:

$$\min_{\mathbf{x}} \left[\left(\underline{f}_0 - \sum_{j=1}^{N} \underline{c}_{j0} x_j \right)^2 + \left(\bar{f}_0 - \sum_{j=1}^{N} \bar{c}_{j0} x_j \right)^2 + \left(\underline{f}_1 - \sum_{j=1}^{N} \underline{c}_{j1} x_j \right)^2 + \left(\bar{f}_1 - \sum_{j=1}^{N} \bar{c}_{j1} x_j \right)^2 \right], \quad (13)$$

where:

$$\underline{f}_0 = \min_{\mathbf{x}} \left[\sum_{j=1}^{N} \underline{c}_{j0} x_j \right], \quad \bar{f}_0 = \min_{\mathbf{x}} \left[\sum_{j=1}^{N} \bar{c}_{j0} x_j \right], \quad (14)$$

$$\underline{f}_1 = \min_{\mathbf{x}} \left[\sum_{j=1}^{N} \underline{c}_{j1} x_j \right], \quad \bar{f}_1 = \min_{\mathbf{x}} \left[\sum_{j=1}^{N} \bar{c}_{j1} x_j \right]. \quad (15)$$

The compromise objective functions (10), (11) and (13) are square functions, and therefore, for these functions, solving an optimisation task of charge burden calculation requires allowing for a system of constraints (2), (3) or (5), respectively, and using a method of square programming.

Practical application of the task of charge burden calculation for charge materials characterised by fuzzy chemical composition and uncertain unit prices should be (in the author's opinion) limited to the case in which the fuzzy price is simulated by a rectangular membership function. Assuming one interval of changes in the unit price of each charge material seems to be sufficient in management of the chain of foundry supplies. The use of more complex functions to describe unit prices considerably expands the compromise functions and makes programming of procedures creating these functions automatically difficult (though not impossible). An example of calculations given below shows the possibility of using in optimisation task a rectangular model of fuzziness of the unit prices.

5.4. Example of charge burden calculation for a fuzzy chemical composition (trapezoidal model) and fuzzy price (rectangular model)

Table 2 gives fuzzy parameters (chemical composition, unit price) of charge materials taken into consideration in this optimisation task. In the example of calculation no additional constraints have been assumed as regards the content of each charge component.

Because of fuzzy unit prices quoted in this example in the form of an interval of their changes (in a rectangular version of the membership function), it is necessary to solve, first, the task of minimising the objective function to a form:

$$f_0 = 2.60x_1 + 2.40x_2 + 2.20x_3 + 5.00x_4 + 4.20x_5 + 4.70x_6 \quad (16)$$

and next the task of minimising the objective function to a form:

$$f_1 = 2.70x_1 + 3.40x_2 + 2.90x_3 + 5.60x_4 + 4.60x_5 + 5.10x_6 \quad (17)$$

under the following constraints:

TABLE 2

Chemical composition and unit price of charge materials and assumed chemical composition of charge for calculation of example

Charge component	Chemical composition, %			Price
	C	Si	Mn	PLN/kg
x_1	(0.22;0.24;0.26;0.29)	(0.26;0.29;0.34;0.40)	(0.15;0.17;0.21;0.25)	2.60÷2.70
x_2	(3.4;3.5;3.6;3.75)	(1.6;1.8;1.9;2.1)	(0.64;0.75;0.86;0.98)	2.40÷3.40
x_3	(3.2;3.25;3.32;3.38)	(1.73;1.84;1.97;2.05)	(0.35;0.38;0.39;0.45)	2.20÷2.90
x_4	(0.05;0.1;0.12;0.15)	(64.0;64.3;64.7;65.0)	(0.25;0.32;0.34;0.35)	5.00÷5.60
x_5	(89.0;90.0;91.0;91.5)	(0.0;0.0;0.0;0.0)	(0.0;0.0;0.0;0.0)	4.20÷4.60
x_6	(0.4;0.44;0.52;0.57)	(0.3;0.4;0.5;0.6)	(81.0;81.6;82.0;82.5)	4.70÷5.10
CHARGE	**3.0÷3.2**	**1.4÷1.6**	**0.6÷0.8**	

$$\begin{cases} 0.22x_1 + 3.4x_2 + 3.2x_3 + 0.05x_4 + 89.0x_5 + 0.4x_6 \geqslant 3.0 \cdot 100 \\ 0.24x_1 + 3.5x_2 + 3.25x_3 + 0.1x_4 + 90.0x_5 + 0.44x_6 \geqslant 3.0 \cdot 100 \\ 0.26x_1 + 3.6x_2 + 3.32x_3 + 0.12x_4 + 91.0x_5 + 0.52x_6 \leqslant 3.2 \cdot 100 \\ 0.29x_1 + 3.75x_2 + 3.38x_3 + 0.15x_4 + 91.5x_5 + 0.57x_6 \leqslant 3.2 \cdot 100 \\ 0.26x_1 + 1.6x_2 + 1.73x_3 + 64.0x_4 + 0x_5 + 0.3x_6 \geqslant 1.4 \cdot 100 \\ 0.29x_1 + 1.8x_2 + 1.84x_3 + 64.3x_4 + 0x_5 + 0.4x_6 \geqslant 1.4 \cdot 100 \\ 0.34x_1 + 1.9x_2 + 1.97x_3 + 64.7x_4 + 0x_5 + 0.5x_6 \leqslant 1.6 \cdot 100 \\ 0.4x_1 + 2.1x_2 + 2.05x_3 + 65.0x_4 + 0x_5 + 0.6x_6 \leqslant 1.6 \cdot 100 \\ 0.15x_1 + 0.64x_2 + 0.35x_3 + 0.25x_4 + 0x_5 + 81.0x_6 \geqslant 0.6 \cdot 100 \\ 0.17x_1 + 0.75x_2 + 0.38x_3 + 0.32x_4 + 0x_5 + 81.6x_6 \geqslant 0.6 \cdot 100 \\ 0.21x_1 + 0.86x_2 + 0.39x_3 + 0.34x_4 + 0x_5 + 82.0x_6 \leqslant 0.8 \cdot 100 \\ 0.25x_1 + 0.98x_2 + 0.45x_3 + 0.35x_4 + 0x_5 + 82.5x_6 \leqslant 0.8 \cdot 100 \\ x_1 + x_2 + x_3 + x_4 = 100 \end{cases} \tag{18}$$

A solution of the task of minimising the objective function (16) under constraints (18) is:

$$\begin{cases} x_1 = 38.73\% \\ x_2 = 0.0\% \\ x_3 = 59.28\% \\ x_4 = 0.50\% \\ x_5 = 1.14\% \\ x_6 = 0.35\% \\ f_0 = 240.06 \end{cases} \tag{19}$$

The task of minimising function (17) with the system of constraints (18) has the following solution:

$$\begin{cases} x_1 = 94.49\% \\ x_2 = 0.0\% \\ x_3 = 0.0\% \\ x_4 = 1.88\% \\ x_5 = 3.13\% \\ x_6 = 0.50\% \\ f_1 = 282.60 \end{cases} \tag{20}$$

For plotting a compromise objective function resulting from the relationship (10), only the values of f_0 from (19) and f_1 from (20) are needed. The fractions of individual charge components given in relationships (19) and (20) are significantly different. This is important inasmuch as in the case of solutions not differing much from each other (only unit price changes), plotting of compromise function makes no sense.

The definition of a compromise objective function is as follows:

$$\begin{aligned} f = [240.06 - (2.60x_1 &+ 2.40x_2 + 2.20x_3 + \\ &+5.00x_4 + 4.20x_5 + 4.70x_6\ )]^2 + \\ + [282.60 - (2.70x_1 &+ 3.40x_2 + 2.90x_3 + \\ &+5.60x_4 + 4.60x_5 + 5.10x_6\ )]^2 . \end{aligned} \tag{21}$$

The task of charge burden calculation for fuzzy parameters of the charge materials (chemical composition and unit price) consists in determination of the value of vector x, which minimises the compromise objective function (21) under constraints (18).

Having introduced the above data to a computer program (computing the algorithm of square programming) and having performed the respective computation procedure, the results shown in Figure 2 are obtained.

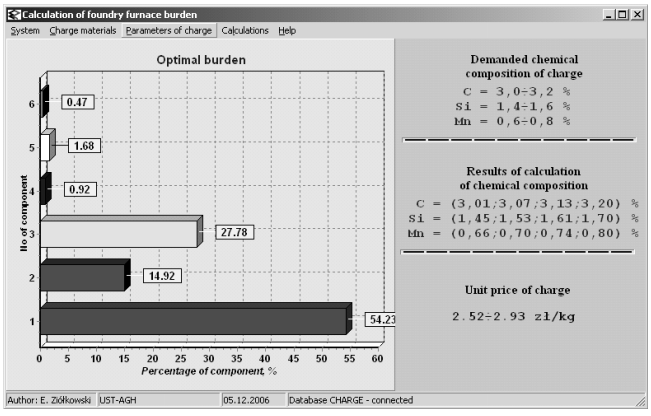


Fig. 2. Output of charge burden calculations

The calculated charge burden is optimal for the whole range of changes in unit prices of the individual charge materials. Choosing this burden (according to Figure 2) will be much "safer" than it would be if only average unit prices of the individual charge components were taken into consideration.

6. Summary

The methods of charge burden calculation described here, considerably extend our knowledge (foundry problems – in particular, but also science in general) about optimising of production processes. Through model approach to the tasks coresponding to various situations occurring in technological process, the operation of charge burden calculation can be aided by computers and included in the system of automatic control of devices which prepare charge burden for foundry furnaces. In integrated system of production planning and preparation, these methods may prove to be useful in searches for a best strategy to manage the chain of supplies of new charge materials.

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