A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 56 2011 Issue 3
DOI: 10.2478/v10172-011-0067-z

D. SARAMAK*

OPTIMAL PRODUCTION OF ELECTROLYTIC COPPER DETERMINED BY THE CONCENTRATION AND DISTRIBUTION OF COPPER CONCENTRATES TO SMELTERS ON THE EXAMPLE OF KGHM

OPTYMALIZACJA PRODUKCJI MIEDZI ELEKTROLITYCZNEJ POPRZEZ SYSTEM WZBOGACANIA I DYSTRYBUCJI KONCENTRATÓW DO HUT NA PRZYKŁADZIE KGHM

The issues of copper production strategy for the Polish copper industry are presented in the article. The problem is considered within the scope of the ore extraction, ore concentration and metallurgical processing of the copper concentrate and the main aim is to produce the specific amount of copper, which generates maximum technological and economic benefits. In order to determine the strategy of production for a company an optimisation approach may be applied with the target function defined as either the maximization of metal recovery or the maximization of profit. The real operating conditions are implemented into the model, together with existing limitation resulting from the concentration technology and metallurgical treatment. The problem verification was possible with using of non-linear programming theory. Two variants were considered: the first for lower production costs, and the second for low stock market prices. The presented optimal strategies determine the copper grades of concentrates as well as the system of concentrates distribution between three smelters. Generally smelters should receive concentrates with higher copper grades for low stock market metal prices and for low processing costs the mass of concentrates delivered to smelters should be higher together with decreased the copper grades.

Keywords: optimization, copper production strategy, non-linear programming, process control, modelling

W pracy przedstawiono zagadnienia dotyczące strategii produkcji miedzi elektrolitycznej dla polskich warunków produkcji miedzi w układzie kopalnia-zakład przeróbczy-huta miedzi. Problem jest rozpatrywany głównie z punktu widzenia hut miedzi, ale determinowany jest jakością i ilością koncentratów miedziowych wyprodukowanych przez zakłady przeróbcze. Celem jest osiągniecie maksymalnych efektów technologicznych i ekonomicznych. Podejście optymalizacyjne zakłada zbudowanie odpowiedniego modelu ekonometrycznego z funkcja celu zdefiniowaną jako maksymalny odzysk metalu oraz uwzględnieniem istniejących realnych ograniczeń technologicznych Weryfikacja modelu jest możliwa z wykorzystaniem oprogramowania działającego w oparciu o teorię programowania nielinowego. W artykule przedstawiono analizę dla dwóch wariantów produkcji miedzi: niskich kosztów produkcyjnych oraz niskich cen giełdowych miedzi. Aby maksymalizować funkcję celu w zaprezentowanym modelu, w okresie niskich cen produkcji huty powinny otrzymywać mniej koncentratów o wyższej zawartości miedzi, natomiast przy niskich giełdowych cenach miedzi, huty powinny przerabiać większe ilości koncentratu o zmniejszonej zawartości miedzi.

1. Introduction

KGHM Polish Copper S.A. has integrated systems of copper production, starting from extracting the ore, through the concentration process, to the metallurgical treatment and pure metal production. Thanks to a such structure it is possible the optimization and effective control of the whole process in the mine–processing plant–copper smelters system, and also the copper production planning with respect to existing realities.

The company comprises the following integral parts: three mines: ZG Lubin, ZG Polkowice, and ZG Rudna (denoted as mine 1, mine 2 and mine 3 respectively), three concentrator plants: OZWR Lubin, OZWR Polkowice and OZWR Rudna (denoted as concentrator 1, concentrator 2 and concentrator 3 respectively) as well as three smelters: HM Legnica, HM Głogów I and HM Głogów II (denoted as smelter 1, smelter 2 and smelter 3 respectively). The final product quality and quantity is considered from the point of view of the metallurgical stage (electrolytic copper), but the concentration stage is the place in the whole copper production circuit, in where the efficiency of the whole process is preliminary

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, DEPARTMENT OF MINERAL PROCESSING AND ENVIRONMENTAL PROTECTION, FACULTY OF MINING AND GEOENGINEERING, 30-059 KRAKÓW, 30 MICKIEWICZA AV., POLAND

TABLE 1

The average content of copper, base cupriferrous minerals, and lithologic composition of ores processed in individual concentrator plants [2]

	Concentrator 1	Concentrator 2	Concentrator 3
Content of copper in ore, %	1,3	2,0	2,2
Content of major coupriferrous minerals, %			
chalcocite	52	80	70
bornite	38	18	25
chalcopyrite	10	2-4	4-5
Lithologic composition, %			
sandstone	60	8	56
carbonate	25	75	33
shale	15	17	11

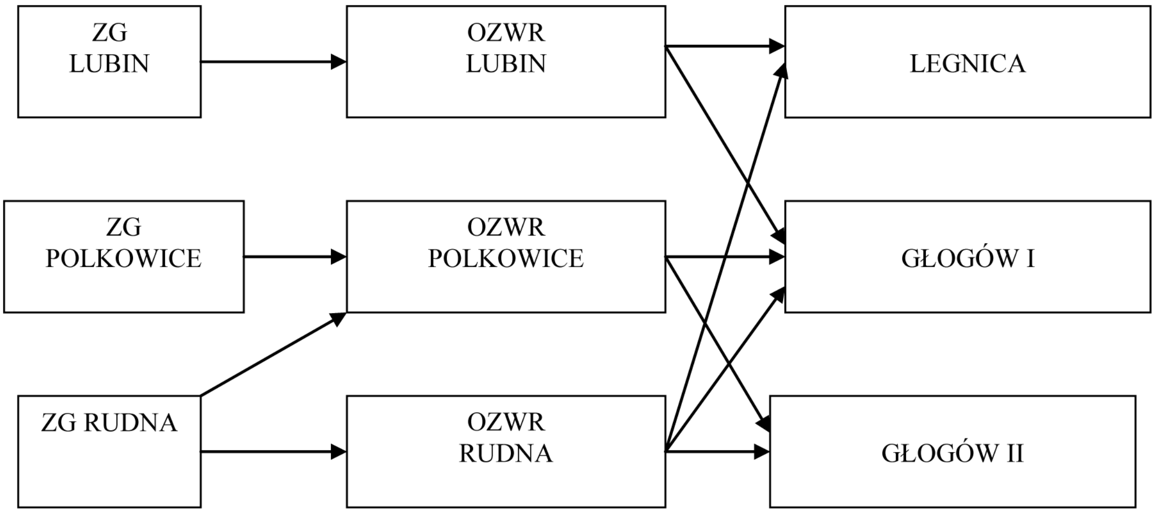

Fig. 1. Material mass flows among individual units of the company

determined. It is caused by proper adjustment of the concentration technology applied in each concentrator plant, especially the parameters of the enrichment process.

Ores processing in the plants are a mixture of three lithologic types: sandstone, carbonate and slate and include, besides of copper, also silver as the valuable component.

The concentration process for different types of ore has it's reflection in the processing technology operating in each individual processing plant. Metallurgical treatment of copper is achieved by the use of two technologies: a flash smelting process (applied in Legnica) and a suspensional one (applied in Głogów I and Głogów II), which is adapted to the selectivity profile of the enriched ores. The main difference between above technologies results from different exergy sources [3]. The material flows system for the company is presented in Figure 1.

Suitable planning of copper production should therefore consider all mass flows among the individual units: from individual mines, through the concentrator plants to the smelters, with regarding the technological limitations as well as the production capacities of individual concentrator plants and smelters.

2. Technological and economic optimizational models

Proper planning of production strategy for copper is connected with the maximization of technological effects (the maximum copper recovery – Z_1) or economic ones (the maximization of profit – Z_2). For the copper production system presented, that is: mine – processing plant – copper smelters, the generalized model, in which the target function is the maximization of copper recovery, can be written as follows:

$$Z_1 = \sum_{i=1}^{3}\sum_{j=1}^{3} M_i \cdot \gamma_j \cdot \beta_j - \sum_{i=1}^{3}\sum_{j=1}^{3} M_i \cdot \left(100 - \gamma_j\right) \cdot \vartheta_j \tag{1}$$

at presence of limitations: $\beta_j > \beta_{min}$,

β_{min} – the minimum copper grade in the concentrate, required by the conditions of the metallurgical treatment

i – number of mines in the whole copper production system; i = 1, 2, 3,
j – number of the concentrator plant; j = 1, 2, 3,
M_i – mass of ore extracted in mine number i,
γ_j – yield of the concentrate transported from processing plant number j,
β_j – copper grade in the concentrate transported from concentrator plant number j,
ϑ_j – copper grade in tails obtained from the enrichment of individual ores.

It is also possible to analyze such optimizational approach by using the function of metal losses [10].

The target function from the point of view of economic optimization is defined as maximization of the company's profit. To attain maximum profit, each concentrator plant has to produce a defined amount of the concentrate with respective copper grade β, and to deliver it for metallurgical treatment to the suitable smelter [7]. It is also essential to determine a suitable content for concentrate mixtures, to the treatment in individual smelters, with regarding the existing technological limitations and the ore properties. The target function has the following form:

$$P = \begin{pmatrix} \sum_{j=1}^{3}\sum_{k=1}^{3} Q_{jk}\beta_j\gamma_j\varepsilon_k P_{Cu} + \\ + \sum_{j=1}^{3}\sum_{k=1}^{3} Q_{jk}(a_j\beta_j + b_j)\gamma_j\varepsilon'_k P_{Ag} \end{pmatrix} - \\ - \begin{pmatrix} \sum_{j=1}^{3} Q_j MC + \sum_{j=1}^{3} Q_j\gamma_j PC_j + \\ + \sum_{j=1}^{3}\sum_{k=1}^{3} Q_{jk}\gamma_j\beta_j HC_k\varepsilon_k \end{pmatrix} \tag{2}$$

where:
j, γ_j, β_j, ϑ_j – denotations as in formula (1)
k – number of smelters in the copper production system; k = 1, 2, 3
γ_{jk} – yield of the concentrate processed in concentrator plant number j, treated in smelter number k,
ε_k – metallurgical recovery of copper for specific smelter,
ε'_k – metallurgical recovery of silver for specific smelter,
a_j, b_j – parameters of equation (14) describing relationships between copper and silver grades in concentrates,
Q_j – mass of the ore processed in a specific concentrator plant,
P_{Cu} – price of copper at world metal stock market,
P_{Ag} – price of silver at world metal stock market,
MC – mining costs,
PC_j – processing costs for a specific plant,
HC_k – metallurgical processing costs for specific smelter.

In model (2), the distribution of concentrates into a specific smelter should be also taken into consideration. The structure of the concentrates distribution to the smelters results mainly from the quality of produced concentrates and technology of metallurgical treatment and is determined by the following formula:

$$\frac{\sum_{j=1}^{3}\sum_{k=1}^{3} \gamma_j Q_{jk} \cdot r_{jk}}{\sum_{j=1}^{3}\sum_{k=1}^{3} \gamma_j Q_{jk}} = 1 \tag{3}$$

where: $\sum_{j=1}^{3}\sum_{k=1}^{3} r_{jk} = 1$, and

r_{jk} – fraction of the whole concentrate enriched in concentrator plant number j, transported into smelter number k.

Concentrates are distributed among the three smelters according to formulas (4) – (10), which define the fixed range proportions of the mixtures transported to the smelters:

a) Concentrates delivered to smelter 1 (Legnica):
– from concentrator plant 1 (Lubin)

$$0,3 \leqslant \frac{\gamma_1 Q_{11} \cdot r_{11}}{\sum_{j=1}^{3} \gamma_j Q_{j1}} \leqslant 0,4 \tag{4}$$

– from concentrator plant 3 (Rudna)

$$0,6 \leqslant \frac{\gamma_2 Q_{31} \cdot r_{31}}{\sum_{j=1}^{3} \gamma_j Q_{j1}} \leqslant 0,7 \tag{5}$$

– from concentrator plant 2 (Polkowice) no concentrate is delivered

b) concentrates delivered to smelter 2 (Głogów I)
– from concentrator plant 1 (Lubin)

$$0,25 \leqslant \frac{\gamma_1 Q_{12} \cdot r_{12}}{\sum_{j=1}^{3} \gamma_j Q_{j2}} \leqslant 0,4 \tag{6}$$

– from concentrator plant 2 (Polkowice)

$$0,2 \leqslant \frac{\gamma_2 Q_{22} \cdot r_{22}}{\sum_{j=1}^{3} \gamma_j Q_{j2}} \leqslant 0,35 \tag{7}$$

– from concentrator plant 3 (Rudna)

$$0,3 \leqslant \frac{\gamma_3 Q_{32} \cdot r_{32}}{\sum_{j=1}^{3} \gamma_j Q_{j2}} \leqslant 0,4 \tag{8}$$

c) concentrates delivered to smelter 3 (Głogów II)
– from concentrator plant 2 (Polkowice)

$$0,4 \leqslant \frac{\gamma_2 Q_{23} \cdot r_{23}}{\sum_{j=1}^{3} \gamma_j Q_{j2}} \leqslant 0,5 \tag{9}$$

– from concentrator plant 3 (Rudna)

$$0,5 \leqslant \frac{\gamma_3 Q_{33} \cdot r_{33}}{\sum_{j=1}^{3} \gamma_j Q_{j3}} \leqslant 0,6 \tag{10}$$

– from concentrator plant 1 (Lubin) no concentrate is delivered to smelter 3

3. Restrictive and supplementary conditions

In order to determine the optimum of the target function (2), yields γ should be determined as the one variable function, namely $\gamma = \gamma(\beta)$, together with the relationships between the copper and silver grades in concentrates, for each concentrator plant. The yield can be obtained from the mass balance equation (11) [9], which is illustrated by the hyperbolical paraboliod (Fig. 2):

$$100\alpha = \gamma\beta + (1-\gamma)\vartheta \tag{11}$$

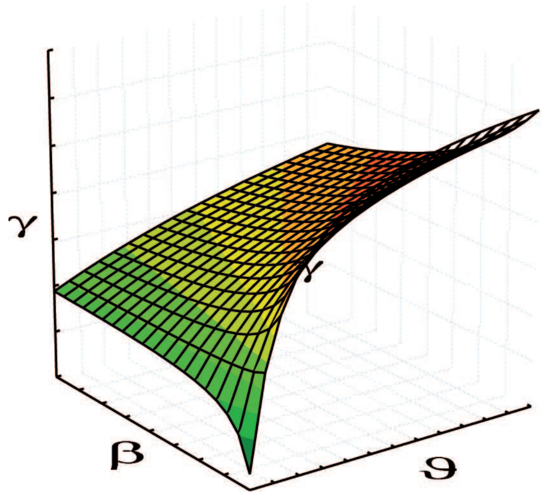

Fig. 2. Functional relationship of yield γ from β and ϑ for fixed value α

The location of the mentioned surface in the 3D co-ordinate system depends on the α value, the paraboloid generally can be placed higher or lower together with respective increase or decrease in α value. The real course of the enrichment process is illustrated by some curve lying on that surface, which takes into account the mineralogical composition of the ore as well as the selectivity of the mineral grains (Fig. 3).

There are two essential points on the hyperbole (Fig. 3), describing the concentration process [5]: A = (α, 100) and B = (100, α). Point A, in which the yield equals 100%, ($\beta = \alpha$), describes the case where the concentration process does not take place. Point B describes a theoretical perfect concentration – the final product consists solely of the whole copper, for this case the value of yield equals α. On the curve (Fig.3) a point can also be determined halfway between A and B with co-ordinates (β_t, γ_t), which describes the perfect process of mechanical concentration, where all cupriferous sulphides contained in the ore pass to the concentrate. After suitable calculations we obtain the relationship (12)

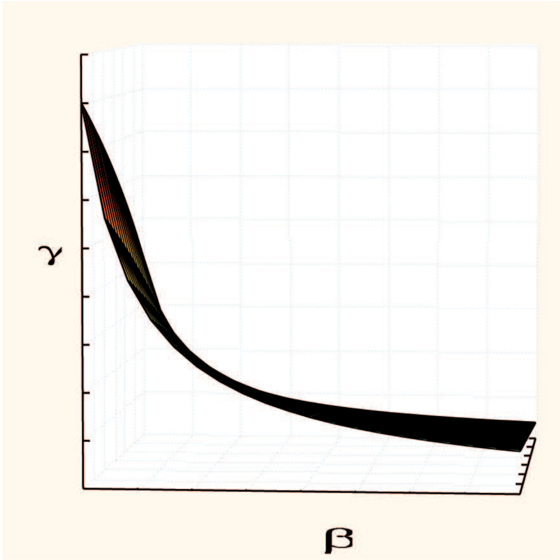

Fig. 3. Projection of the surface described by formula (13) on (γ;β) plane

$$\gamma = \frac{100 \cdot \alpha}{\beta} \tag{12}$$

or a generalized form of the hyperbola (12):

$$\gamma = \frac{a}{\beta} + b \tag{13}$$

where a, b – coefficients.

Investigations of the selectivity of copper and silver run in the Non-Ferrous Metals Institute in Gliwice (Poland) [4,11] as well as laboratory experiments conducted on copper and silver selectivity show, that for a specific ore a linear dependence exists between the content of copper and silver in the enrichment products. For the copper concentrate the above dependence can be denoted in following formula:

$$\beta_{Ag} = a_j \cdot \beta_j + b_j \tag{14}$$

where:
β_{Ag} –silver grade in the concentrate,
β_j – copper grade in the concentrate,
a_j,b_j – coefficients.

Additional limitations are connected with the type of metallurgical treatment technology in specific smelter. Depending the smelter, the concentrates are processed in, their quality cannot be lower than β_{min_j} denoting the minimum copper grade in the concentrate.

4. Obtained solution – the model verification

The presented model can possibly be solved by means of the software basing on non-linear programming theory. The real operating limitations resulting from the concentration technology and from economic factors (stock market metal prices) must naturally be taken into consideration in the model in order to obtain the solution on the satisfying level of accuracy. The main aim of solving the presented model is then an effective determination of variables existing in the target function, taking into consideration defined limitations, process flowsheets as well as the world stock market metal prices. In this solution the model will produce an optimal set of parameters' values which characterize the optimal operation of the considered copper production circuit and the best system of their distribution among individual smelters. Analysis of obtained solutions provides the opportunity to work out the operating strategy for the plants: the proper copper production planning as well as the maintaining the technological indices of the concentration process [6,8].

Various alporhitus can be used in order to solve optimisation models [12, 13]. Calculations were proceeded by means of GAMS software. The program works on the basis of non-linear programming principles, with CONOPT algorithm as a tool [1]. The calculations were performed for two variants of copper production conditions. Variant 1 was prepared for lower production costs, while the second variant reflects the situation of the recent financial crisis (low metal stock market prices). Significant variation in copper stock market prices (Fig. 4) and the Polish currency (PLN) can be also observed for the second modeling variant.

The modeling results for both variants are presented in Table 2. For comparison, real operating data for the KGHM plant in the respective period of time was also presented. Profit values for all variants were presented in the Polish currency units (PLN).

Examining the Table 2 one can see, that the modeling results are close to the real operating conditions. The model even admit the higher concentrate copper grades for concentrator plant 1 and 3 than the real production. In the optimal solution the concentrate distribution system for variant 1 should be as follow:

a) concentrates to smelter 1 should be delivered in following proportions: 57.4% from concentrator 3 and 42.6% from concentrator 1
b) concentrates to smelter 2 should be delivered in following proportions: 43.8% from concentrator 1, 25.5% from concentrator 2 and 30.7% from concentrator 3
c) concentrates to smelter 3 should be delivered in following proportions: 37.1% from concentrator 2 and 62.9% from concentrator 3

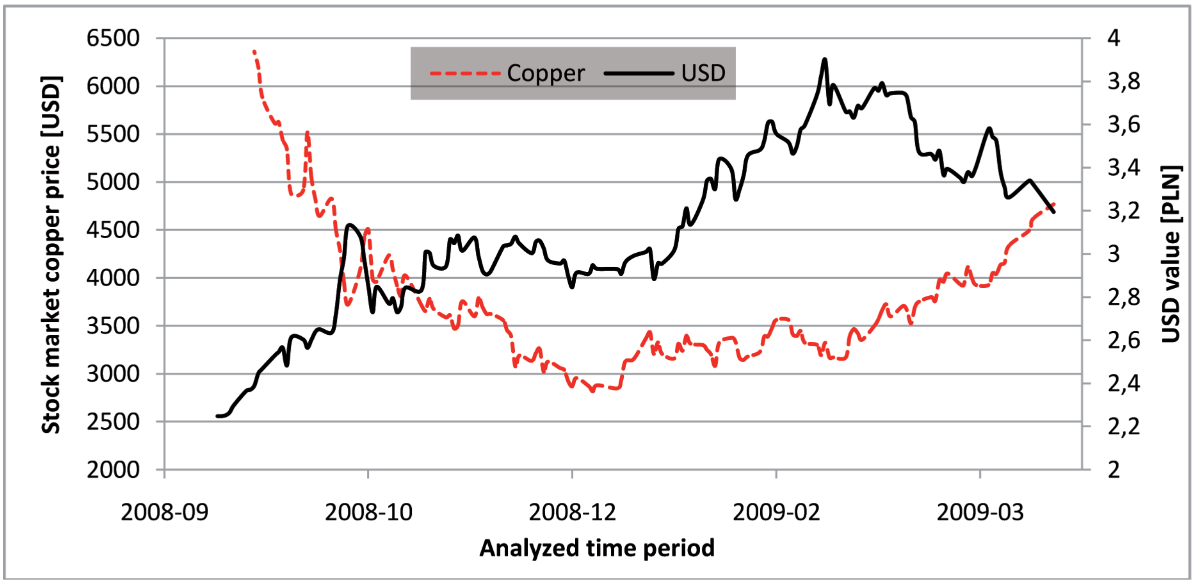

Fig. 4. Copper stock market prices and the USD value

TABLE 2

Modeling results and respective real operating ones for variant 1 and 2

Variant 1		
	Modeling results	Real operating results
Main technological indices [%]		
β_1 (concentrator 1)	19.3	17.8
β_2 (concentrator 2)	25.9	27.1
β_3 (concentrator 3)	31.3	30.6
γ_1 (concentrator 1)	5.2	no data
γ_2 (concentrator 2)	6.5	no data
γ_3 (concentrator 3)	5.8	no data
Target function value profit (P) [x10^6PLN]	765	570
Variant 2		
	Modeling results	Real operating results
Main technological indices [%]		
β_1 (product from concentrator 1)	20.5	no data
β_2 (product from concentrator 2)	26.3	no data
β_3 (product from concentrator 3)	32.4	no data
γ_1 (concentrator 1)	4.8	no data
γ_2 (concentrator 2)	6.4	no data
γ_3 (concentrator 3)	5.5	no data
Target function value profit (P) [x10^6PLN]	563	521

The optimal concentrate distribution system for variant 2 should be as follow:

a) concentrates to smelter 1 should be delivered in following proportions: 56.8% from concentrator 3 and 43.2% from concentrator 1
b) concentrates to smelter 2 should be delivered in following proportions: 45.0% from concentrator 1, 25.5% from concentrator 2 and 29.5% from concentrator 3
c) concentrates to smelter 3 should be delivered in following proportions: 36.6% from concentrator 2 and 63.4% from concentrator 3

Analyzing the proportion of the concentrate distribution system it is easy to notice that in the feed to smelter Legnica and Głogów I the product from concentrator 1 (Lubin) should have the higher proportion in variant 2 than in variant 1. The feed to smelter 3 (Głogów II) should have also the higher proportion of concentrate from Rudna in second variant.

In the Fig. 5 the optimal quality of the concentrate as a function of copper stock market prices is presented. Values of copper stock market are presented on horizontal axis in percentages. A one hundred percent value relates to the one in first variant of modeling. On the basis of these results we may notice, that the amount of processed concentrate should be higher in the period of high metal stock market prices and lower in the period of lower metals' prices. This is connected with the production of concentrate with a lower copper grade (at high stock market prices), what causes the higher metal recovery (leading to a reduction in paymetal loss).

The relationship between the concentrate copper grade and the silver stock market prices (Fig.6) has similar course like for copper. Similarly like on Fig. 5 the one hundred percent value relates to the one in first variant of modeling. The influence of silver stock market prices on the optimal copper grade has much lower importance than the copper, however. Costs of metallurgical treatment have the inverse proportional course to the concentrate copper grade. For the higher metallurgical costs the production of concentrates with higher copper grade has the economic justification (Fig. 7).

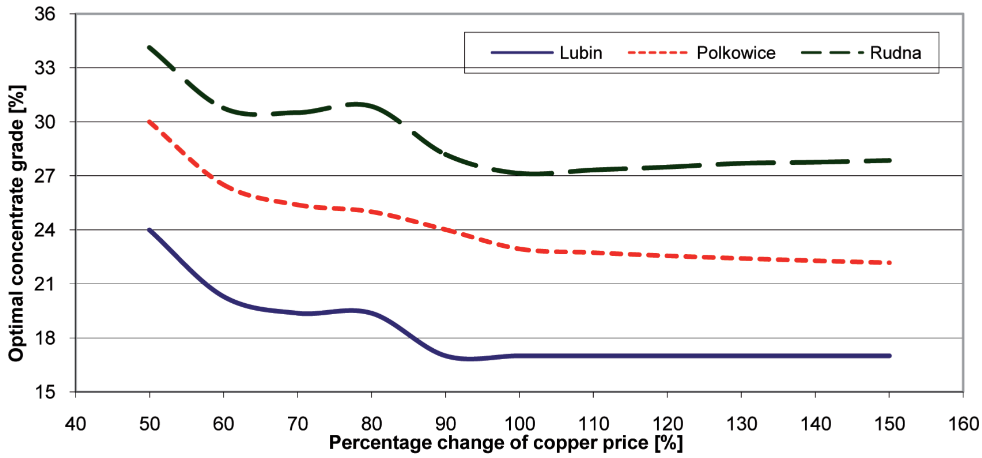

Fig. 5. Relationship between optimal quality of the concentrate and market price of copper

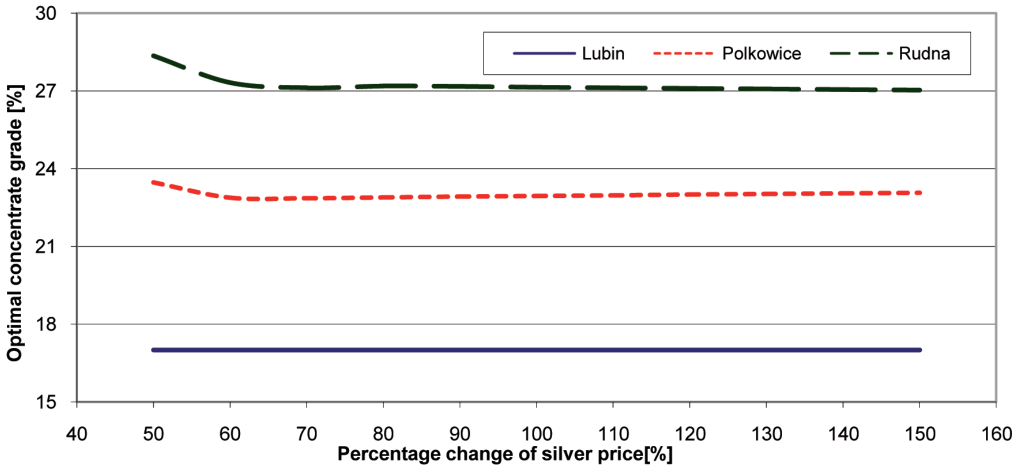

Fig. 6. Relationship between optimal quality of the concentrate and market price of silver

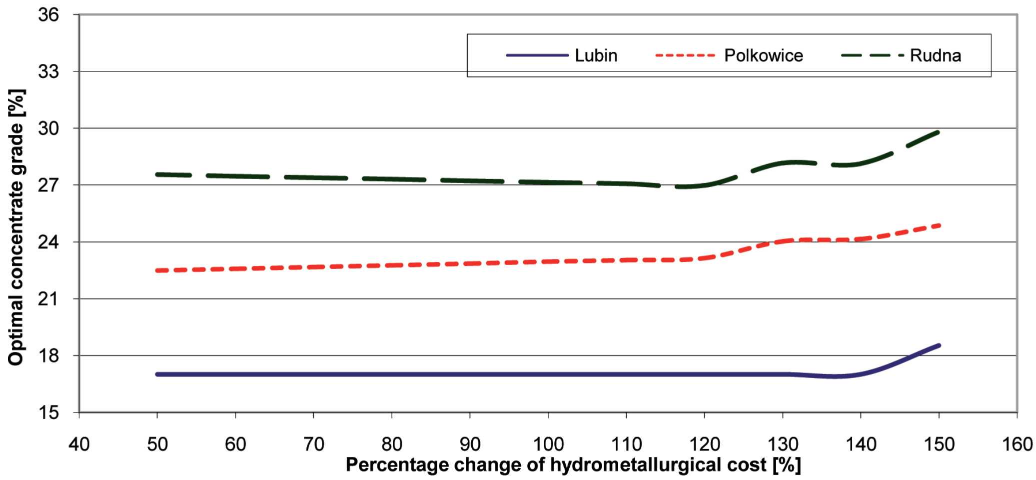

Fig. 7. Relationship between optimal quality of the concentrate and metallurgical costs

The value of target function for both modelling variants is close to the real operating results. Even though a considerable changeability of stock market copper prices was observed, especially for the second variant, the results are close to the reality. It proves that the concentration technology in KGHM is close to the optimum and the model was property worked out.

5. Summary

The model designed is convergent to the reality and the obtained modeling results are close to those obtained for the real operating conditions of the plant (Table 2). Generally, we can determine the following operating strategies according to the obtained results:

– lower copper stock market prices – higher copper grade in the concentrate

– lower processing costs (metal recovery can be higher) – lower copper grade in the concentrate

We suppose that for higher stock market prices the copper grade of the produced concentrates should be lower due to the paymetall loss is also lower. The higher processing costs allow for production of concentrates with rather lower copper grades, but the possible decision should be taken after considering the price – cost relationship. Table 3 sums up the possible optimal decisions determined on the basis of obtained modeling results.

TABLE 3

Possible strategies (copper grades of produced concentrates) resulting from the relationship between processing costs and stock market metal prices

	Low processing costs	High processing costs
High stock market prices	low	Rather lower, but depends on the relationship price – cost
Low stock market prices	Rather higher but depends on the relationship price – cost	high

The analysis can be also performed with regard to the decisive variables being either the world exchange price of silver, or the cost of metallurgical treatment, or finally the character of the ore processed. Such analyses are the subject of constant research and this matter will be the object of subsequent investigation.

The implementation of the presented model into industrial operation is a rather complex issue. Three basic problems should be solved:

– the adjustment of the technological circuit for production of the concentrates with copper grade according to the regime determined by the model.
– the suitable concentrates distribution system between the smelters, according to the formula (3) and also with the production of concentrate mixtures.
– updating coefficients in formulas (13) and (14) because of the ore variability.

The implementation is then a separate, mainly logistic, task being in charge of the management of KGHM.

REFERENCES

[1] Drud, A.S. CONOPT – a large scale GRG code. ORSA Journal of computing, 207-216 (1992).
[2] Monograph of KGHM, CUPRUM (1996).
[3] J. Norwisz, B. Boryczko, A. Hołda, Z. Kolenda, Thermo-ecological cost analysis of shaft and flash smelting process of copper production – general approach, Archives of Metallurgy and Materials **54**, 1, 153-159 (2008).
[4] Report of Non-Ferrous Metals Institute no. 5035/94: Odzysk Ag w procesie wzbogacania miedzi z rudy z obszaru górniczego ZG Polkowice. Gliwice (1994).
[5] D. Saramak, Optymalizacja uzysku metali w Zakładach Wzbogacania Rud w KGHM "Polska Miedź" S.A., doctoral thesis AGH UST, Cracow (2004).
[6] D. Saramak, Symulacyjne modele pracy układów technologicznych przeróbki surowców, Górnictwo i Geologia **3**, 3, 49-60 (2008).
[7] B. Skorupska, D. Saramak, Methods of determination the optimal quality of the concentrate according to chosen technological and economic criteria. Mineral Resources Management **21**, 4, 5-20 (2005).
[8] B. Skorupska, D. Saramak, T. Tumidajski, A. Wieniewski, The influence of concentrate's quality on the economic efficiency of multi-plant mining and metallurgical company, Mineral Resources Management **23**, 4, 39-52 (2007).
[9] T. Tumidajski, D. Saramak, Wielowymiarowa analiza wskaźnika oceny przebiegu procesów inżynierii mineralnej opartych na prawie zachowania masy. Mineral Resources Management **18**, 2, 77-90 (2002).
[10] T. Tumidajski, D. Saramak, B. Skorupska, Chosen indexes of technological assessment of mineral resources enrichment processes as a function of the concentrate's quality. XII Balkan Mineral Processing Congress, Delphi, Grece (2007).
[11] A. Wieniewski, N. Kubacz, B. Spalińska, Odzysk srebra w procesie wzbogacania polskich rud miedzi. XXVI Scientific Conference of Mineral Processing, Cracow **13** (1994).
[12] T. Domański, A. Bokota, Numerical Models of Hardening Phenomena of Tools Steel base on the TTT end CCT Diagrams, Archives of Metallurgy and Materials **56**, 2 (2011.
[13] A. Kulawik, A. Bokota, Modelling of Heat Treatment of Steel with the Movement of Coolant, Archives of Metallurgy and Materials **56**, 2 (2011).

Received: 10 February 2011.