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NUMERICAL SIMULATION OF TURBULENCE FLOWS IN SHEAR LAYER

SYMULACJA NUMERYCZNA PŁYNIĘCIA TURBULENTNEGO W WARSTWIE ŚCINANEJ

For various problems of continuum mechanics described by the equations of hyperbolic type, the comparative analysis of scenarios of development of turbulent flows in shear layers is carried out. It is shown that the development of the hydrodynamic instabilities leads to a vortex cascade that corresponds to the development stage of the vortices in the energy and the inertial range during the transition to the turbulent flow stage. It is proved that for onset of turbulence the spatial problem definition is basic. At the developed stage of turbulence the spectral analysis of kinetic energy is carried out and the Kolmogorov "-5/3" power law is confirmed.

Keywords: mathematical modeling, shear layers, energy cascade, turbulence

W pracy dokonano analizy porównawczej rozwoju przepływu turbulentnego w warstwach ścinanych dla wielu problemów dotyczących mechaniki ośrodków ciągłych opisanych przez równania typu hiperbolicznego. Wykazano, że rozwój niestabilności hydrodynamicznych prowadzi do cyklicznych zawirowań podczas przejścia do stadium przepływu turbulentnego. Udokumentowano, że dla wyjaśnienia mechanizmu pojawienia się turbulencji kluczowe znaczenie ma przestrzenna analiza problemu. Przeprowadzono analizę widmową energii kinetycznej dla zaawansowanego stadium przepływu turbulentnego oraz potwierdzono wartość wykładnika "-5/3" w równaniu Kołmogorowa.

1. Introduction

Despite the variety of turbulent flows in nature, they still remain the least studied and are the subject of intense experimental and theoretical investigations. To analyze the structure and development of the turbulent motion it is very important to study processes linking the emergence of turbulence and transition to the stage of developed large-scale turbulent flow.

The concept of an energy cascade goes back to Richardson's idea [1] of the eddy turbulence structure scaling down to microscales at which viscous dissipation dominates. This concept was used by Kolmogorov and Obukhov in their studies [2, 3], which led to the well known spectral structure of the energy cascade: at sufficiently high Reynolds numbers, the fluctuation energy density distribution over wave numbers is divided into a near range of low numbers (energy range), where energy is generated basically due to the instability of large eddies; a far range of high numbers (viscous range), where energy dissipates into heat through small scale fluctuations; and an inertial spectral range lying in between, where energy is neither generated and nor dissipate but is transferred from smaller to larger wave numbers [2]. According to [2], this exchange depends weakly on the instability of the large scale flow and the Reynolds number determined by the original flow. The underlying physical process is the loss of stability of a sequentially forming basic flow and the formation of a new velocity field with a finer eddy structure.

Below, this scenario of Richardson–Kolmogorov–Obukhov turbulence development is applied to a shear layer subject to a constant external forcing (Kolmogorov's problem). In this problem, turbulence arises due to Kelvin–Helmholtz instabilities. This type of instability is most frequently encountered in mixing layers, where, due to the high velocity gradients, the influence of viscosity and the walls on the general characteristics of flow macrostructures is negligible.

Formally inviscid flows are described by the Euler equations that, in contrast to the Navier -Stokes equations, determine the turbulent flow at infinite high Reynolds numbers. Direct numerical simulation of the turbulent flows based on the Navier-Stokes equations is currently possible only for Reynolds numbers less than 30000. In this case energy, inertial and viscous intervals are simulated. In practically important problems of turbulent flows, the Reynolds number exceeds 10^6. This shows that the simulation of turbulent flows on the basis of the Euler equations does not mean a loss of accuracy or approximate description than simulation based on the Navier- Stokes equations. It is a good mechanism for the investigation of the energy and the inertial range of turbulence. While modeling using large eddy simulation (LES), subgrid turbulence models may yield different results with respect to the mean values of higher moments, but for the quantities de-

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termined by large eddies, numerical simulation results differ slightly [4].

Through research on the basis of the Euler equations, we consider the problems of appropriate energy and the inertial range of the energy spectrum of turbulence without considering the effects of viscosity. It is necessary to utilize models that take into account the viscosity and described by the Navier -Stokes equations [5] when you deal with the calculation of viscous flows in the range where the energy is dissipated as heat through small-scale pulsations. However, this kind of modeling is only possible in the range of Reynolds numbers less than 30000, and requires the use of tera- and petaflops supercomputers. At the same time simulation based on the Euler equations can serve as a basis to predict the results in the energy and inertial intervals, including Kolmogorov's spectrum in the homogeneous isotropic turbulence.

This paper shows that the development of the turbulence occurs when the inertial terms and the pressure field in the equations of motion begin to form large structures and within vortices appear. Further evolution of the flow is developed via large eddies and generating high-frequency part of the spectrum. The main aim here is to study the overall dynamics of the development and the nature of the turbulent flow through a cascade of vortex instabilities.

Due to extreme complexity and nonlinearity of turbulent flows, an adequate tool to study them is the numerical simulation [5].

The monotone dissipative- stable difference schemes with positive operator [6], well-proven for calculating large-scale flows, were used for modeling of the turbulence in shear layers in the inviscid case. These schemes are of the second order of accuracy for smooth solutions and being monotone, do not use any artificial viscosity, nor smoothing, nor procedures that limit the flow, and often used in modern computational fluid dynamics. Numerical simulation was performed utilizing the MPI parallel programming interface to model systems with up to 10^6 computational cells.

2. Mathematical model and the numerical method

The model of compressible inviscid gas is used for the modeling. Starting point for the constructing numerical calculation schemes is the complete system of the Euler equations(in case of the Kolmogorov's problem we used the complete system of the Euler equations with right-hand side) written in divergence form in Cartesian coordinates [7]. This is the equation for the density of the medium:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

The equations for the three components of the momentum density:

$$\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho \vec{v} u) = -\frac{\partial p}{\partial x}$$

$$\frac{\partial (\rho \mathrm{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \mathrm{v}) = -\frac{\partial p}{\partial y}$$

$$\frac{\partial (\rho w)}{\partial t} + \nabla \cdot (\rho \vec{v} w) = -\frac{\partial p}{\partial z} - \rho g$$

and the equation for the density of total energy

$$\frac{\partial (\rho e_0)}{\partial t} + \nabla \cdot ((\rho e_0 + p)\vec{v}) = -\rho g w$$

Where t is the time, *(x, y, z)*are the coordinates; $\vec{v} = (u, v, w)$is the velocity vector; is the density; *g* is the force of gravity, $e_0 = e + \mathrm{v}^2/2$ is the specific total energy, *e* is specific internal energy. The equation of the state is required to close the system of equations. In this paper the equation of the state of a perfect gas is considered. All calculations were performed in the SI measurement system.

In numerical simulations we have used the monotone dissipative-stable difference schemes with positive operator that does not require the introduction of subgrid turbulence and special filters for the simulation of free fully developed turbulence [5]. The proposed method is a generalization of the explicit hybrid scheme [6]. This scheme has the second order of accuracy for smooth solutions and being monotonous, does not use any artificial viscosity or smoothing, or procedures that limit the flow (flux limiter), often used in some schemes of the computational fluid dynamics. It is a combination of schemes with oriented and central differences for the linear transport equation. For this scheme the monotonicity condition is performed strictly, i.e. any monotonic set of function values at the grid remains monotonous through the time step. In this technique, switching between schemes with central and oriented differences is performed separately for the each characteristic and depends on the sign of the corresponding characteristics and sign of a single additional parameter.

Our computational model does not account viscosity and the surface tension, however, the very design of the scheme with the requirement of monotonicity provides some nonlinear dissipative mechanism that ensures the damping of short harmonics. In other words, the harmonics with a wavelength less than a certain effective wavelength are retarded. This is confirmed by our calculations. Obviously, approximately equals to a few steps of numerical finite-difference grid. With this technique, we have performed an extensive series of instabilities calculations [8-11]. The results show a good agreement with theory and experiment.

3. The problem

The initial stage of the onset of turbulence in three- dimensional compressible inviscid shear flow with and without external power is studied. The integration domain is shaped as 3D parallelepiped in Carthesian coordinates XYZ (**Fig. 1**). We investigate the evolution of the structure of shear flow of width H=1 in case when the initial velocity along Y coordinate linearly changes from -V to V inside a shear layer. Boundary and initial conditions are identical for both shear layer without external force and Kolmogorov's problem. Boundary conditions along X and Y coordinates are periodic, along Z coordinate we used conditions of impermeability. We use initial conditions for the velocity along x-direction and z-direction inside the shear layer in form of one Fourier mode:

$$w = ampl * \sin(2\pi y) * \cos(\pi z)$$

and velocity along x-direction is equal

$$u = ampl * \sin(2\pi y) * \cos(\pi z).$$

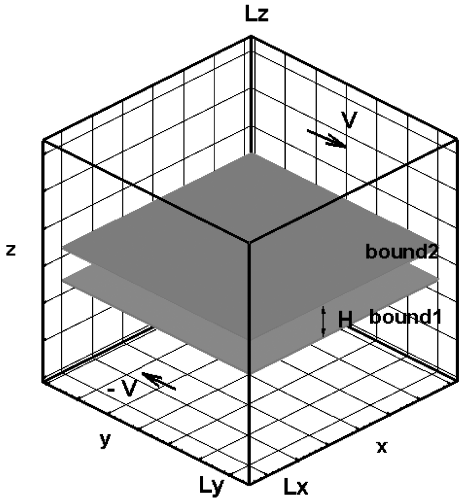

Fig. 1. Computational domain and the basic flow parameters

Let us describe the spectral characteristics of the developing flow. Consider the spectral representation of kinetic energy over the vertical coordinate in the middle of the domain. By analyzing the arising turbulence patterns, we traced the formation of a stable spectral segment for velocity component fluctuations. For the inertial range, the following estimates for the fluctuation energy distribution $E(k)$ of the velocity field [2] were derived from similarity considerations:

$$E(k) \approx \Theta^{2/3} k^{-5/3},$$

where k is the wave number and Θ is the energy dissipation rate per unit mass. The Kolmogorov "–5/3" power law was obtained relying on the theory of dimensions, and it must hold for turbulent motion under the applicability conditions for the Kolmogorov–Obukhov theory, namely, if the turbulent motion is homogeneous and isotropic in the inertial range. For inhomogeneous and anisotropic actual flows, the applicability conditions are violated and the spectrum in the inertial range can differ from Kolmogorov's one.

4. Results

In this work we consider initial stage of the onset of the turbulence for both problems. It is shown that in case of a problem without affecting the constant force of the vortex cascade develops as follows: the evolution of the flow at the beginning demonstrates a quasi-two-dimensional nature (the onset of instability begins with the formation of large-scale vortex). However, evolving further, the large-scale vortex changes its shape with time and finally gets destroyed (**Fig. 2**). Formation of the vortex cascade in Kolmogorov's problem is following. In contrast to the shear layer, the flow loses its stability due to instabilities in the form of a comb across the excited surface. Over time, this comb is stretched due to the formation of new instabilities. Large structure is formed implicitly. This

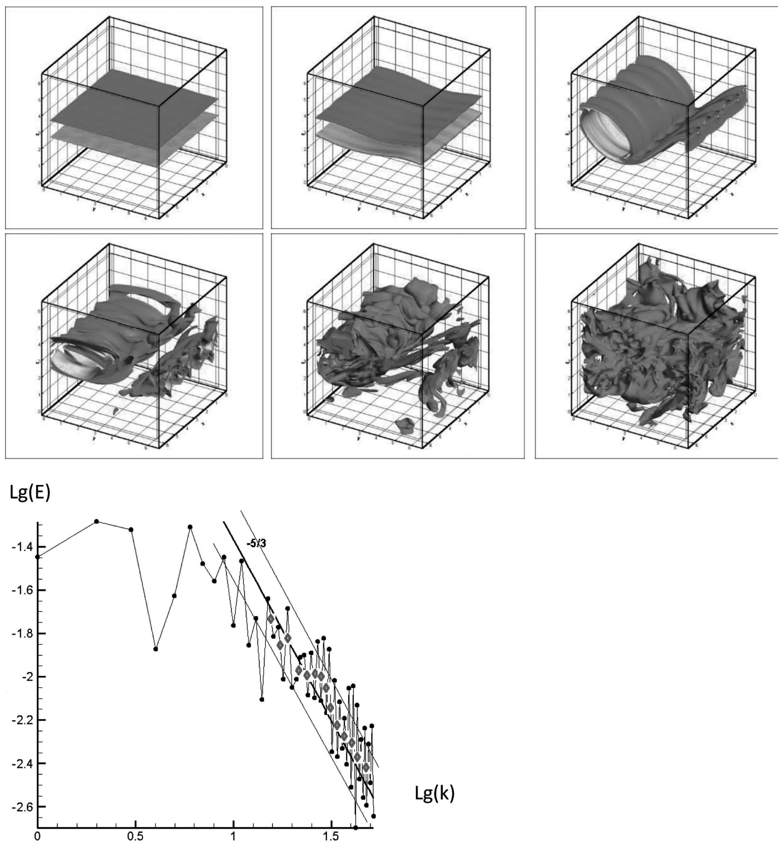

Fig. 2. Vortex cascade. Equiscalar surfaces of the density and spectrum of kinetic energy for shear layer problem. Time moments t=0,5,7,10,12,17

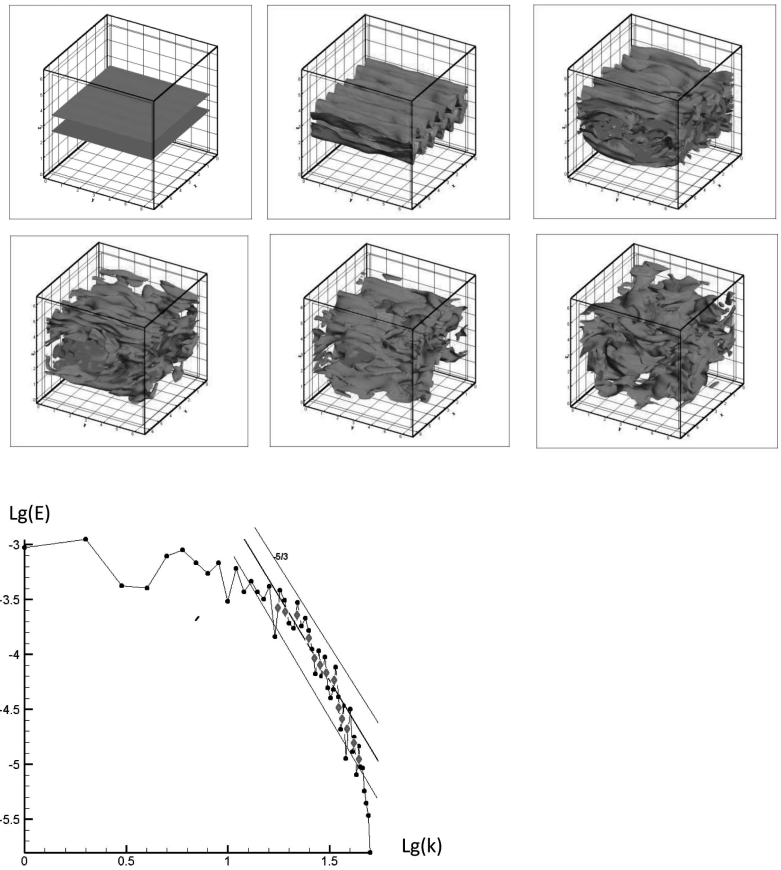

Fig. 3. Vortex cascade. Equiscalar surfaces of the density and spectrum of kinetic energy for Kolmogorov's problem. Time moments t=0,5,7,10,12,17

structure finally also gets destroyed. It leads to the collapse into smaller vortexes. Thus, in this case the vortex cascade is also exist (**Fig. 3**).

Special attention is given to energy spectrum of kinetic energy. It is shown that as the flow transforms to the turbulent phase there emerge pulsations of velocity of various scale which leads to the formation of a vortex cascade. Decomposition of kinetic energy on wave number reveals correspondence with the energy spectrum of Kolmogorov-Obukhov and the Kolmogorov "-5/3" power law for both problems. Based on the numerical results, an analysis of the spectral exponent α in the shear layer problem

$$\alpha = -\frac{5}{3} \pm 0.3,$$

produced that. Thus, up to 30% accuracy, the numerical results suggest that the energy spectrum contains an inertial range.

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